ORIGINAL ARTICLE

Time-fractional Burgers equation for dust acoustic waves in a two different temperatures dusty plasma

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Abstract The reductive perturbation method has been used to derive the Burgers equation for dust acoustic shock waves in unmagnetized plasma having electrons, singly charged ions, hot and cold dust species with Boltzmann distributions for electrons and ions in the presence of the cold (hot) dust viscosity coefficients. The time-fractional Burgers equation is formulated using Euler-Lagrange variational technique and is solved using the variational-iteration method. The effect of time fractional parameter on the behavior of the shock waves in the dusty plasma has been investigated.

Keywords Burgers equation · Shock waves · The time-fractional Burgers equation

1 Introduction

The wave propagation in dusty plasmas has received much attention in the recent years because of its vital role in understanding different types of collective processes in space environments, namely, lower and upper mesosphere, cometary's tails, planetary rings, planetary magnetosphere, interplanetary spaces, interstellar media, etc. (Mendis and Rosenberg 1994; Verheest 1996; Shukla and Mamun 2002, 2003; Sayed and Mamun 2007 and Pakzad 2009). In dusty plasma, the dust grains may be charged negatively by plasma electron and ion currents or positively by secondary electron emission, UV radiation, or thermionic emission etc.

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Theoretical Physics Group, Physics Department, Faculty of Science, Mansoura University, Mansoura 35516, Egypt e-mail: emadshewy@yahoo.com (Whipple 1981). Due to the higher thermal velocity of electron than ion, the dust grains usually acquire a negative charge in low temperature laboratory dusty plasma (Barkan et al. 1994). However, in a laboratory Q machine, positively charged dust grains may be produced by replacing the plasma electrons with negative ions whose thermal velocity is smaller than that of positive ions (D'Angelo 2004). In most of the previous investigations of dust acoustic waves (DAWs), the dust has been assumed to be cold or hot. Recently, Akhtar et al. (2007) studied the large amplitude DAWs in unmagnetized two types of dust fluids (one is cold and the other is hot) in the presence of Boltzmann ions and electrons.

On the other hand, a medium with dissipative properties supports the existence of shock waves instead of solitons. The dust fluid dissipation can be caused by Landau damping, dust-dust collision, dust fluid viscosity and dust charge fluctuation, which would modify the wave structures (Nakamura and Sarma 2001; Singh and Rao 1998; Popel et al. 2000; Nakamura et al. 1999). Under appropriate conditions, shock waves can be propagated in the system. The effects of dissipation caused by kinematic viscosity on the propagation of solitary wave structure are discussed by Nakamura and Sarma (2001). The KdV and Burgers equations appear quite naturally in plasma physics. The dispersion of the ion-acoustic wave balances the positive nonlinearity to form solitons (Schamel 1975). For a wave possesses a positive nonlinearity like the ion-acoustic wave, damping like collisional damping is needed to form the shock waves which results in the Burgers equation. Recently, Nakamura et al. (1999) observed shock waves in unmagnetized dusty plasma. They showed that the development of the shock is due to KdV-Burgers (KdV-B) equation. Rahman et al. (2007) investigated the effects of ion-fluid temperature on