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ORIGINAL ARTICLE

Exact self-similar Bianchi II solutions for some scalar-tensor theories

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Abstract We study how may behave the gravitational and the cosmological "constants", (G and Λ) in several scalar-tensor theories with Bianchi II symmetries. By working under the hypothesis of self-similarity we find exact solutions for three different theoretical models, which are: the Jordan-Brans-Dicke (JBD) with $\Lambda(\phi)$, the usual JBD model with potential $U(\phi)$ (that mimics the behavior of $\Lambda(\phi)$) and the induced gravity (IG) model proposed by Sakharov and Zee. After a careful study of the obtained solutions we may conclude that the solutions are quite similar although the IG model shows some peculiarities.

Keywords Exact self-similar Bianchi II · Scalar-tensor theories · Time-varying constants

1 Introduction

Current observations of the large scale Cosmic Microwave Background (CMB) suggest to us that our physical universe is expanding in an accelerated way. Such observations (Bernadis et al. 2000; Hanany et al. 2000; Balbi et al. 2000), indicate that the universe is dominated by an unidentified "dark energy" (DE) and suggests that this unidentified dark energy has a negative pressure (Perlmutter et al. 1998, 1999; Riess et al. 1998; Garnavich et al. 1998). This last characteristic of the dark energy points to the vacuum energy or cosmological constant Λ, as a possible candidate for dark energy. From the theoretical point of view, it is convenient to consider the cosmological constant as a dynamical quantity in order to solve the so called coincidence and fine tuning problems.

In the same way other observations have pointed out a possible variation of the gravitational constant G (D'Innocenti et al. 1996 and Umezu et al. 2005). In particular, observations of Hulse–Taylor binary pulsar (Bisnovatyi-Kogan 2006; Damour et al. 1988), Type Ia supernova observations (Riess et al. 1998). For an extensive review see Uzan (2010).

We have several theoretical models that consider both constants as variable with respect to the cosmic time. Such theories are: modified General Relativity (MGR), modified scalar cosmological models (MST) and several scalar-tensor theories (STT). The MGR and MST have a drawback since in them the variation of G and Λ are introduced in an ad hoc manner. Nevertheless we consider that the STT are the best models to study the variation of G and Λ since they have been deduced form variational principles and where the time dependence can occur in a natural way, without any new assumption or modification of the theory. This class of models has received a renewed interest in recent times, for two main reasons: First, the new inflationary scenario as the extended inflation has a scalar field that solves several problems present in the old theories. Secondly, string theories and other unified theories contain a scalar field which plays a similar role to the scalar field of the STT. The scalar-tensor theories started with the work of P. Jordan in 1950 (Jordan 1955). A prototype of such models was proposed by Brans and Dicke in 1961 (Brans and Dicke 1961). Their aim for presenting this model was to modify Einstein's theory in such a way as to incorporate the so-called "Mach principle". These theories have been generalized by Bergmann (1968), Nordtverdt (1970) and Wagoner (1970). For a recent review of this class of theories we refer to Fujii and Maeda (2003) and Faraoni (2004).

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