Exact solutions for the incompressible viscous magnetohydrodynamic fluid of a rotating-disk flow with Hall current

M. Turkyilmazoglu

Mathematics Department, University of Hacettepe, 06532-Beytepe, Ankara, Turkey

ABSTRACT

The focus of the present study is to obtain exact solutions for the flow of a viscous hydromagnetic fluid due to the rotation of an infinite disk in the presence of an axial uniform steady magnetic field with the inclusion of Hall current effect. In place of the traditional von Karman's axisymmetric evolution of the flow, the rotational non-axisymmetric stationary conducting flow is taken into consideration here, whose governing equations allow an exact solution to develop bounded everywhere in the normal direction to the wall.

The three-dimensional equations of motion are treated analytically yielding derivation of exact solutions, which differ from those of corresponding to the classical von Karman's conducting flow. Making use of this solution, analytical formulas for the angular velocity components, for the current density field as well as for the wall shear stresses are extracted. The critical peripheral locations at which extrema of the local skin friction occur are also determined. It is proved from the analytical results that for the specific flow the properly defined thicknesses decay as the magnetic field strength increases in magnitude, approaching their hydrodynamic value in the limit of large Hall numbers.

Interaction of the resolved flow field with the surrounding temperature is further analyzed via the energy equation. The temperature field is shown to accord with the dissipation function. According to the Fourier's heat law, a constant heat transfer from the disk to the fluid occurs, though it increases by the presence of magnetic field, the increase is slowed down by the Hall effect eventually reaching its hydrodynamic limit.

1. Introduction

A substantial interest in rotating magnetohydrodynamic viscous fluid flow motion has been witnessed in the past few decades, due to the reason that it provides practical applications to many engineering areas, for instance, pumping and levitation of liquid metals.

von Karman's swirling hydrodynamic viscous flow [1] is a well-documented classical problem in fluid mechanics, which has several technical and industrial applications. The original problem raised by von Karman, which is the most studied by researchers in the literature, is the viscous flow motion induced by an infinite rotating disk where the fluid far from the disk is at rest. Then the problem is generalized to include the case where the fluid itself is rotating as a solid body far above the disk. Another generalization is to consider the viscous flow between two infinite coaxial rotating disks. All these problems and also stability issues are attacked, theoretically, numerically and experimentally, by many researchers amongst many others, such as [2–13]. However, all of these results are either numerical or analytical-numerical. Moreover, [14] established the existence of an infinite set of solutions to the flow of two infinite parallel plates rotating with the same angular velocity about an axis. Extension to porous plates was given in [15,16], refer also to [17]. Additionally, heat transfer problem over a rotating disk was also studied, see for instance [18,19].

The cases when an exact solution for the Navier–Stokes equations can be obtained are of particular importance in investigations to describe fluid motion of the viscous fluid flows. However, since the Navier–Stokes equations are non-linear in character, there is no known general method to solve the equations in full, nor the superposition principle for non-linear partial differential equations does work. There are some well-known exact unidirectional or parallel shear flows, a few sample cases contain the steady Poiseuille and Couette flow. In addition to this, there are exact cylindrically symmetric solutions with closed plane streamlines, see for instance [20,21]. Further exact solutions that we already know possess certain feature of the fluid motion, such as rectilinear motion, motions of the duct flows, axisymmetric flows and stagnation flows on plate with slip, etc., see for instance, the works of [22–24].

E-mail address: turkyilm@hotmail.com