

Contents lists available at ScienceDirect

# International Journal of Non-Linear Mechanics



journal homepage: www.elsevier.com/locate/nlm

# Moving wedge and flat plate in a power-law fluid

# Anuar Ishak<sup>a,\*</sup>, Roslinda Nazar<sup>a</sup>, Ioan Pop<sup>b</sup>

<sup>a</sup> School of Mathematical Sciences, Faculty of Science and Technology, Universiti Kebangsaan Malaysia, 43600 UKM Bangi, Selangor, Malaysia <sup>b</sup> Faculty of Mathematics, University of Cluj, R-3400 Cluj, CP 253, Romania

#### ARTICLE INFO

Article history: Received 1 December 2008 Accepted 7 April 2011 Available online 17 April 2011

*Keywords:* Boundary-layer Dual solutions Moving wedge Non-Newtonian power-law fluid

### 1. Introduction

## Many important industrial fluids are non-Newtonian in their flow characteristics and are referred to as rheological fluids. One particular class of materials of considerable interest is that in which the effective (or apparent) viscosity depends on the rate of shearing or, crudely speaking, on the flow rate. Most particulate slurries (China clay and coal in water, sewage sludge, inks, etc.) and multiphase mixtures (oil-water emulsions, gas-liquid dispersions such as froths and foams, butter, etc.) are non-Newtonian fluids, as well as melts and solutions of high-molecularweight naturally occurring and synthetic polymers. Other examples of systems displaying a variety of non-Newtonian characteristics include pharmaceutical formulations (cosmetics and toiletries, paints, synthetic lubricants), biological fluids (blood, synovial fluid, saliva, etc.) and foodstuffs (jams, jellies, soups, marmalades, etc.). Because such fluids have more complicated equations that relate the shear stresses to the velocity field than Newtonian fluids have, additional factors must be considered in examining various fluid mechanics and heat transfer phenomena. Numerous excellent books (Schowalter [1], Bird et al. [2], Chhabra [3], Slattery [4], etc.) and review articles (Irvine Jr. and Karni [5], Andersson and Irgens [6], Ghosh et al. [7]) summarizing the current status-of-the-art of research of non-Newtonian fluids are now available.

In recent years, interest in the boundary-layer flows of non-Newtonian fluids has increased. Rajagopal et al. [8] looked at the boundary-layer flows of fluids of second grade, and later, Rajagopal et al. [9] studied the Falkner–Skan flows of a homogeneous incompressible fluid of second grade. One of the main areas of

### ABSTRACT

The steady boundary-layer flow of a non-Newtonian fluid, represented by a power-law model, over a moving wedge in a moving fluid is studied in this paper. The transformed boundary-layer equation is solved numerically for some values of the involved parameters. The effects of these parameters on the skin friction coefficient are analyzed and discussed. It is found that multiple solutions exist when the wedge and the fluid move in the opposite directions, near the region of separation. It is also found that the drag force is reduced for dilatant fluids compared to pseudo-plastic fluids.

© 2011 Elsevier Ltd. All rights reserved.

interest is the boundary-layer behavior of a non-Newtonian fluid past a surface in motion relative to either a stationary or moving fluid. This situation represents a different class of boundary-layer problem which has a solution substantially different from that of the boundary-layer flow over a fixed surface and is an important type of flow occurring in a number of material processing applications. Examples of processes include continuous casting, plastic forming, bonding, annealing and tempering, heat treatment, cooling of an infinite metallic plate in a cooling bath, the boundarylaver along a liquid film in condensation processes and a polymer sheet or filament extruded continuously from a dye, or a long thread traveling between a feed roll and a wind-up roll, and many others. Owing to the non-linear equation of state for non-Newtonian fluids, the resulting governing equations are much more complex in form than their Newtonian counterparts and therefore, numerical solutions have often been sought. Sakiadis [10] was the first to recognize this new class of problems for a viscous and incompressible (Newtonian) fluid and presented analyses for the two-dimensional and axisymmetric boundary-layer flows past a surface moving with a uniform velocity (rigid surface). Due to the entrainment of the ambient fluid, this flow situation represents an intrinsically different class of boundary-layer flows which have a substantially different type of solution as compared to the case of a static surface. It seems that Andersson and Dandapat [11] were the first who have obtained similarity solutions for the non-Newtonian flow of a power-law fluid past an impermeable stretching surface, while Andersson et al. [12] have extended this problem to the case of a magnetohydrodynamic flow of a power-law fluid over a stretching sheet in the presence of a uniform transverse magnetic field. Liao [13] has recently presented a very nice analytic solution of the magnetohydrodynamic flow of a non-Newtonian power-law fluid over an impermeable stretching sheet using the homotopy analysis method. The boundary-layer flow of a non-Newtonian power-law fluid with injection on a permeable semi-infinite flat

<sup>\*</sup> Corresponding author. Tel.: +603 8921 5756; fax: +603 8925 4519. *E-mail address:* anuarishak@yahoo.com (A. Ishak).

<sup>0020-7462/\$ -</sup> see front matter  $\circledcirc$  2011 Elsevier Ltd. All rights reserved. doi:10.1016/j.ijnonlinmec.2011.04.020