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On large elastic deformation of prestressed right circular annular cylinders

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ABSTRACT

We formulate and study inflation, extension and twisting of prestressed cylindrical shells that are isotropic in the stress free configuration. We establish that if the prestresses vary only radially in the annular cylinder then a deformation field of the form $r=\hat{r}(R)$, $\theta=\Theta+\Omega Z$, $z=\lambda Z$ is possible in annular cylinders made of any incompressible material and find sufficient conditions for the deformation to be possible when made of compressible materials. When the material is capable of undergoing large elastic deformations and has a non-linear constitutive relation, for the cases studied here, there is up to 26 percent variation in the boundary loads required to engender a given boundary displacement between the prestressed and stress free annular cylinders. On the other hand, the difference in the realized deformation field is only marginal (less than 2 percent). These are unlike the case wherein the material obeys Hooke's law and undergoes small deformations. This study has some relevance to the deformation of blood vessels.

1. Introduction

According to Withers and Bhadeshia [1] almost all manufactured components cannot be intact and stress free at the same instant because of the presence of misfits. These misfits can be intentionally designed as in a shrunk fit shaft or can arise due to the chemical, thermal or inelastic process that the component has been subjected to. The stress field in the configuration used as reference configuration, is called the prestress field. It is common in the literature to refer to what we call as prestress as residual stresses. However, we find that calling prestresses as residual stresses though apt in some cases it is not so for many others. As the nomenclature makes it clear, by residual stresses we understand that they are the stresses remaining at the end of some process that the body has been subjected to. But the stresses in a configuration need not arise only due to the process it can has been subjected to. For example, a body in static equilibrium while subjected to gravitational field cannot be traction free and hence stress free. Therefore, any static configuration used as reference configuration on the surface of the earth will have stresses in them which do not necessarily arise due to some process that they have been subjected to. Moreover since, here we are not interested in the cause for the stresses present in the configuration used as reference, we prefer to call these stresses in the reference configuration as prestresses. However, we do require that the prestress field result in no boundary traction. In other words, here we assume that the prestresses do not arise due to any body forces, like the gravitational field. Even though there exist no material which is inert to gravitational field, we still ignore them. The rationale behind this assumption, which requires validation, is that the prestresses due to gravitational forces would be very small compared to the applied stresses or the prestresses arising due to reasons other than body forces.

It is common practice to ignore the influence of these prestresses in the stress analysis, just because they are difficult to include in the analysis. However, as the design of engineering components becomes less conservative, there is increasing interest in how prestresses affects the mechanical response of the components [1]. Superposition of these prestresses with the service stresses yields fairly accurate results when the material undergoes small deformations and the stress is linearly related to some measure of the strain. However, when the deformations are large and/or the stress is non-linearly related to stretch ratios or strain, one cannot superpose solutions and hence the effect of these prestresses is not known. Hence, here restricting ourselves to materials capable of undergoing large elastic deformations, like rubber and soft biological tissues, we investigate whether

1. The boundary load required to realize a given boundary displacement is the same from a prestressed and stress free configuration.
2. The interior deformation field is the same from a prestressed and stress free configuration for a given boundary condition,

given that the prestresses are such that it does not result in any boundary traction. The importance of the requirement that the

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