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## Magnetic torque attitude control of a satellite using the state-dependent Riccati equation technique

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ABSTRACT

A non-linear attitude control method for a satellite with magnetic torque rods using the statedependent Riccati equation (SDRE) technique has been developed. The magnetic torque caused by the interaction with the Earth's magnetic field and the magnetic moment of torque rods plays a role of the control torque. The detailed equations of motion for this system are presented using angular velocity and quaternions. The SDRE controller is developed for the non-linear systems which can be formed in pseudo-linear representations using the state-dependent coefficient (SDC) method without linearization procedure. The atability regions for the SDRE controlled satellite system are estimated through the investigation of the stability conditions developed for pseudo-linear systems and the application of Lyapunov's theorem. For comparisons, the Linear Quadratic Regulator (LQR) method using the solution of the algebraic Riccati equation (ARE) is also applied to this non-linear system. The performance of the SDRE non-linear control system demonstrates more robustness and stability than the LQR control system when subjected to a wide range of initial conditions.

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## 1. Introduction

A number of approaches for control designs of highly non-linear systems have been examined in past years. These approaches include feedback linearization, adaptive control, gain scheduling, non-linear model predictive control, and sliding mode control methods [1]. Unlike these methods, the state-dependent Riccati equation (SDRE) technique proposed by Cloutier et al. in 1996 [2], is well known as an optimal control method which can be applied for non-linear systems. Recently, the SDRE technique is being applied actively to various fields of non-linear applications [3]: autopilot design [4], guidance law design [5], missile guidance and control system [6], simultaneous state and parameter estimation [7], and control design of various non-linear systems such as a continuously stirred tank reactor [8], an under-actuated robot [9], magnetically levitated ball [10], system with parasitic effects [11], an artificial human pancreas [12], ducted fan [14], benchmark problems [15], etc. Typically, the SDRE technique is also used in the control and estimation of non-linear system related to satellites. A new nonlinear filter, state-dependent Riccati equation filter (SDREF), is introduced by Mracek et al. [16], and applied to angular rate estimation by Harman and Bar-Itzhack [17]. In these two papers, the performance of the SDREF is compared with the performance of the linearized Kalman filter (LKF) and the extended Kalman filter (EKF). Also, Luo and Chu [18] used the SDRE method to achieve a large-angle attitude maneuver control, and French [19] designed control algorithm for rapid, large angle slewing maneuver of a satellite, and it has been implemented on SIMSAT with both thrusters and reaction wheels as control inputs. Stansbery and Cloutier [20] applied the SDRE technique to the development of a controller to maneuver and orient a spacecraft in the proximity of a tumbling target. In this study, they used a simple equation neglecting gravity, and assumed control actuators such as thrusters, rotors and reaction wheels. Attitude control problems based on the SDRE technique also have been applied to satellites with attitude hardware such as internal momentum rotor [21] and reaction wheel [22]. In addition, Irvin [23] proposed the SDRE application for the reconfiguration of satellite formations, and compared with linear quadratic regulator (LQR) and sliding mode controller. Chang et al. proposed attitude control methods in satellite formation flying [24] and satellite tethered formation flying [25]. However, no previous research has been presented on the application of the SDRE technique on attitude control based on only magnetic torque rods.

Small satellites have the advantage in low cost missions due to their minimum hardware configurations and low-power consumption. These missions include those for scientific purposes and remote sensing in a solo or formation flying [26,27]. The attitude control method for these small satellites is required to

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