Development of an analytical reference stress stress equation for inner-diameter defected curved plates in tension

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1. Introduction

Generally, the failure of a defected structure is governed by two different failure modes: plastic collapse and fracture. Both modes can be simultaneously investigated using a failure assessment diagram (FAD) as described in some standards and recommended practices, e.g. R6 \cite{1}, BS7910 \cite{2}, FITNET \cite{3}, API RP579 \cite{4}. On the one hand, the calculation of proximity to plastic collapse in a FAD analysis (plotted on the horizontal axis) requires knowledge of a limit load, defined as the collapse load of the structure, assuming a perfectly plastic material. A situation of ‘local collapse’ can be investigated, in which case the limit load corresponds to a collapse of the ligament ahead of the defect. In contrast, ‘global collapse’ refers to the yielding of the entire cross section containing the defect. Completely equivalent to the concept of a limit load is the so-called reference stress. This stress is defined in such a way that, when the limit load is achieved, it reaches the metal’s yield strength. The concept of reference stress assumes a perfectly plastic yielding behaviour. By definition, limit load and reference stress are connected through the following relation \cite{5}:

\[ \frac{\sigma_{\text{ref}}}{\sigma_y} = \frac{P}{P_L} \]  

(1)

where \( \sigma_{\text{ref}} \) is the reference stress, \( \sigma_y \) the yield stress, \( P \) the applied load, and \( P_L \) the limit load. In a FAD diagram, \( \sigma_{\text{ref}}/\sigma_y \) is denoted as \( L_r \) and plotted on the horizontal axis.

On the other hand, the calculation of proximity to fracture in a FAD analysis (plotted on the vertical axis) requires knowledge of the crack driving force, expressed in terms of stress intensity factor \( K \), crack tip opening displacement (CTOD) or \( J \) integral. \( K \) applies to linear-elastic fracture mechanics, whereas CTOD and \( J \) integral are related quantities in elastic-plastic fracture mechanics. To estimate the crack driving force, Ainsworth \cite{5,6} started from Kumar and Shih’s \cite{7} results to obtain an expression for \( J \) integral that requires a reference stress:

\[ J = \frac{K^2}{E} \left( \frac{E\sigma_{\text{ref}}}{\sigma_{\text{ref}}} + \frac{\sigma_{\text{ref}}^3}{2E\sigma_{\text{ref}}^2} \right) \]

(2)

In this expression, \( K \) is the linear-elastic mode-I stress intensity factor, \( E \) is equal to Young’s modulus \( E \) for plane stress, and to \( E/(1 - \nu^2) \) for plane strain, where \( \nu \) is Poisson’s ratio. \( \sigma_{\text{ref}} \) is the reference strain, which corresponds to \( \sigma_{\text{ref}} \) on the stress–strain diagram of the material. A reference stress that corresponds to the

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