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## A global limit load solution for plates with surface cracks under combined end force and cross-thickness bending

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## ABSTRACT

A global limit load solution for rectangular surface cracks in plates under combined end force and cross-thickness bending is derived, which allows any combination of positive/negative end force and positive/negative cross-thickness moment. The solution is based on the net-section plastic collapse concept and, therefore, gives limit load values based on the Tresca yielding criterion. Solutions for both cases with and without crack face contact are derived when whole or part of the crack is located in the compressive stress zone. From the solution, particular global limit load solutions for plates with extended surface cracks and through-thickness cracks under the same loading conditions are obtained. The solution is consistent with the limit load solution for surface cracks in plates under combined tension and positive bending due to Goodall & Webster and Lei when both the applied end force and bending moment are positive. The solution reduces to the limit load solution for plain plates under combined end force and cross-thickness bending when the crack vanishes.

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## 1. Introduction

In a structural integrity assessment, using the R6 procedure [1] for example, a surface defect in a structure may often be simplified to a surface crack in a plate subjected to combined end force and cross-thickness bending. In the current version of R6 [1], global limit load solutions for surface cracks in plates under combined tension and positive bending, i.e. the applied moment tends to open the crack mouth, are given, based on the solutions developed by Goodall & Webster [2] and Lei [3]. However, in structural integrity assessments, sometimes only the cross-thickness stress distribution is available and the applied loads are basically the resultants of the cross-thickness distributed load. This could result in cases such as a strong tensile end force with a negative bending moment or a strong positive moment with a negative end force. Sometimes, both negative end force and negative moment may occur and this may need an assessment only against plastic collapse. The assessment of all these cases requires a general limit load solution for any combination of end force and positive/negative bending moment.

Many limit load solutions for plates with surface cracks have been developed and available in published literatures. Some of the

solutions developed before 1988 for tension or bending moment were included in a review paper written by Miller [4]. Goodall & Webster [2] and Lei [3] developed a global limit load solution for a rectangular surface crack in a plate under combined tension and positive bending, which is adopted by the current R6 procedure. Lei [5] also developed a global limit load solution for a semi-elliptical surface crack in a plate under combined tension and positive bending. A local limit load solution was developed by Sattari-Far and Dillström [6] also for a plate with a surface semi-elliptical surface crack in a plate under combined tension and positive bending, based on elastic-perfectly plastic finite element (FE) analyses. However, none of the available solutions deal with negative end force and bending moment intending to close the crack mouth (negative bending moment). In this paper, general limit load solutions for rectangular surface cracks in plates under combined tensile/compressive end force and cross-thickness positive/negative bending are derived based on the net-section collapse principle [7]. Note that, in R6 [1], a surface defect in a plate may be characterised as a semi-elliptical crack or a through-wall crack, depending on the position and size of the defect. However, in plastic collapse analysis, a semi-elliptical crack is often replaced by a rectangular crack, which circumscribes the original semi-elliptical crack (e.g. Ref. [2]). This simplification leads to a conservative limit load solution. Lei [5] compared the limit load solutions for rectangular and semi-elliptical cracks in plates under combined tension and positive bending and found that the

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