The stress trajectories method for plane plastic problems

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1. Introduction

This work is aimed at further development of numerical methods for stress identification in solids based on the stress trajectory concept. This concept is widely used in photoelasticity for separation of stresses (e.g. Frocht, 1941); also it has recently been developed to address the problem of elastic stress reconstruction in the Earth’s crust from the data on stress orientations (Galybin and Mukhamediev, 2004). However, the stress states in some regions of the crust can better be described by assuming non-elastic behaviour. There are a number of plastic models in geomechanics (see Appendix A) that presume the state of limiting equilibrium. This means that the governing equations consist of the differential equation of equilibrium (DEE) and a criterion that imposes certain algebraic relationships between the stress components. The most common criteria (Amontons, Mohr–Coulomb) assume linear relationships between the normal and shear stresses acting on the yielding plane. Applications of these criteria to the stress identification in the lithosphere have been reported in Fischer (2006), Hieronymus et al. (2008), Stein et al. (1992), Zoback et al. (2002) and Zoback and Zoback (2007). These studies however do not directly use the data on stress orientations in the formulations of corresponding boundary value problems. The present paper suggests a numerical approach capable of constructing plastic solutions based on a general yield criterion and the boundary data on stress orientations. The examples are limited to the plane problems and to the Tresca and Mohr–Coulomb criteria in order to provide comparisons with the previous studies.

The theory of plasticity for the plane bodies in limiting equilibrium can be found in classical texts by Hill (1950) and Sokolovskii (1965). This theory has been applied in numerous engineering applications, such as stability of foundations and slopes (Sokolovskii, 1965), metal forming (Hill, 1950) or stress distributions around cracks (Kachanov, 1971). Mathematically, the theory is represented by the complete system of governing equations that consists of two DEE reflecting a particular yield criterion and the boundary conditions. Numerical scheme is based on the finite difference method in the Cartesian coordinates. It is shown that the system of two partial differential equations (PDE) is of hyperbolic type and therefore it has two distinct and real families of characteristics. They coincide with the slip lines, observed in experiments, emerging when the state of limiting equilibrium is achieved (when the failure envelope touches the Mohr’s circle). From mechanical point of view the slip lines represent a system of two isogonal families of curves which are tangent to the lines of limiting equilibrium. Several boundary value problems (BVP) can be formulated for hyperbolic systems (see Sokolovskii 1965...