An MPEC approach for the critical post-collapse behavior of rigid-plastic structures

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ABSTRACT

This paper presents a numerical method to identify and trace the critical post-collapse response of rigid perfectly-plastic structures. To account for the possibility of multiple equilibrium paths, the critical one is directly identified using the minimum 2nd-order work criterion. Our proposed enhanced sequential limit analysis is formulated as an instance of the challenging class of optimization problems known as a mathematical program with equilibrium constraints (MPEC). This MPEC formulation minimizes the 2nd-order work expression subject to the set of constraints describing the complete complementarity system (in mixed static-kinematic variables) governing simultaneously the two adjacent equilibrium configurations, namely the current one and its neighboring state. We use a nonlinear programming based algorithm, involving relaxation of the complementarity terms, to solve the MPEC. Four numerical examples are provided to illustrate application of the proposed scheme.

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1. Introduction

With the introduction of limit state criteria in design, it has become mandatory to study the behavior of structures well into the material nonlinearity range. Such behavior is also often accompanied by large deformations, the effects of which must be assessed. In view of this, the analysis for the post-collapse behavior of structures is an important task.

This can be accurately carried out, albeit at considerable computational expense, through an evolutive (step-by-step) elastoplastic analysis for the full history of loading. Such a task entails sophisticated numerical integration of the constitutive laws describing the inelastic material behavior (e.g. Comi et al., 1991; Cocchetti and Maier, 2003; Le Van et al., 2003), often coupled with some iterative predictor-corrector type algorithm (e.g. Hellweg and Crisfield, 1998; Tangaramvong and Tin-Loi, 2010). The inherent numerical difficulties are well-known, and special care needs to be taken to ensure convergence and to capture the most critical equilibrium path.

At variance with a step-by-step elastoplastic analysis, the adoption of classical limit analysis provides a “direct” approach which avoids a computationally expensive time-stepping analysis (Kamenjarzh, 1996). Under the basic assumption of rigid perfectly-plastic materials, a limit analysis determines in a single step the load factor at which plastic collapse occurs. Associated with the collapse limit, the kinematic rate variables provide a feasible set of instantaneous nodal velocities of the structure for the current configuration. Mechanically, the plastic strain rates define a feasible collapse mechanism.

To account in some simplified manner for the effect of geometric nonlinearity, these velocities can be further integrated over a finite time step to produce incremental plastic deformations which can be used to update the configuration of the deforming structure for which a new limit analysis can be carried out. By using a cycle of repeated geometric updating and limit analysis, the post-collapse behavior of the structure undergoing large deformations can be traced.

This simple scheme is the genesis of the well-known so-called sequential limit analysis that has been widely used to investigate the post-collapse behavior of various rigid-plastic structures (e.g. De Freitas and Lloyd Smith, 1989; Yang, 1993; Setzberger and Rammerstorfer, 1999; Corradi and Panzeri, 2003, 2004; Leu, 2005, 2008). The results not only provide useful information as to whether the structure stiffens or softens as a result of geometric nonlinearity, but have also been used to assess the energy dissipation of shell-like bumpers (Corradi and Panzeri, 2003, 2004) and, in the case of frames, can furnish key information for the classical approximate assessment of elastoplastic failure loads (e.g. Horne, 1963). The prominent feature of such an approach is that the computation at each finite step is efficient to carry out. Sufficiently accurate responses can be achieved with relatively large step sizes, as compared to those usually employed for an evolutive elastoplastic analysis. Such a methodology is not only simple but also delivers numerical stability irrespective of the complexity of material and geometric nonlinearities involved.

Loss of uniqueness, or more specifically bifurcation of the kinematic solutions (collapse mechanisms) is an issue with the standard sequential limit analysis. The fact that a single collapse load...