Extended truss theory with simplex constraints

F. Kovács

Department of Structural Mechanics, Budapest University of Technology and Economics, Műegyetem rkp. 3, Budapest H-1521, Hungary

Abstract

This paper traces a way of generalization of the classical truss theory: in addition to the kinematic constraint expressing the distance between two nodes connected by a bar element, other similar constraints involving three and four nodes are introduced. Derived from energy principles, a general tangent stiffness formulation is given. Possible mechanical interpretations as well as problems of pre-stressing are also discussed.

Keywords:
Kinematic constraint
Potential energy
Second-order rigidity
Simplex constraint
Stress
Tangent stiffness

1. Introduction

Tangent stiffness formulation of pin-jointed structures has been studied more or less directly in several papers in the recent years. Although there are various approaches to the interpretation of a tangent stiffness matrix, it is common to distinguish three of its components due to (a) material stiffness (Guest, 2006), (b) the modifying effect of member forces and (c) geometrical stiffness (e.g. Przemieniecki, 1968). However, care must be taken with these denominations, as ‘geometrical stiffness’, e.g. in Schenk (2006) is applied for the sum of members (b) and (c), while Tarnai and Szabó (2002) use ‘complementary stiffness’ for the negative of members (b) and (c) together. Guest (2006) refers to members (a) and (b) together as ‘modified material stiffness’ and calls the matrix corresponding to member (c) the ‘stress matrix’. The term stress is also of doubtful usage anyway: in the theory of mathematical rigidity (concerned only with problems of prestress stability without respect to material properties, i.e. members (b) and (c) of the tangent stiffness only), ‘stress’ is applied for a member force over member length (see e.g. Connelly and Whiteley, 1996), while other sources use tension coefficient (e.g. Southwell, 1920) or force density (e.g. Schek, 1974) with the same meaning.

As was emphasized in Guest (2006), different understanding of the same tangent stiffness is mainly due to different approaches of each field of science, as well as to the way of derivation of the tangent stiffness. The main objective of this paper is to provide a potential energy-based formulation of the tangent stiffness and extended definition of ‘stress’ for a generalized truss model using the Hellinger–Reissner principle (Washizu, 1982). The general feature of our model comes from the introduction of a generalized kinematic constraint which we call simplex constraint. The denomination refers to the physical content of such a constraint, that is, it may express the ‘volume’ of an arbitrary simplex instead of a simple length (of a bar); for instance, area of a triangle spanned by three nodes in a two- or higher-dimensional space or volume of a tetrahedron in three- (or, theoretically, higher-) dimensional space. Conditions of applicability of simplices with zero volume (called degenerate simplices hereinafter) are also analysed.

With the help of this generalized truss model, we want to put different modelling techniques (e.g. used for traditional truss structures or for bar structures with sliding connections) in a unified framework, as well as to suggest some other applications. We will follow a similar path as in Tarnai and Szabó (2002) discussing traditional bar-and-joint assemblies or in Kovács and Tarnai (2009) for the spherical adaptation of the same.

The outline is as follows: in Section 2, a brief resume of the Hellinger–Reissner principle and the corresponding tangent stiffness formulation for bar-and-joint assemblies (trusses) is given. Afterwards, Section 3 provides an extension of the previous theory to higher-dimensional cases, whose mathematical background is presented in Section 4, together with some comments on the existence of second-order rigidity in Section 4.4. Section 5 contains illustrative sample problems showing low-level applicability of the simplex constraints, while the closing Section 6 gives the summary of the work done, completed by mentioning some further applications and problems that need investigation.