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Periodic crack problem for a functionally graded half-plane an analytic solution

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ABSTRACT

The plane elasticity problem of a functionally graded semi-infinite plane, containing periodic imbedded or edge cracks perpendicular to the free surface is considered. Cracks are subjected to mode one mechanical or thermal loadings, which are represented by crack surface tractions. Young's modulus, conduction coefficient, coefficient of thermal expansion are taken as exponentially varying functions of the depth coordinate where as Poisson ratio and thermal diffusivity are assumed to be constant. Fourier integrals and Fourier series are used in the formulation which lead to a Cauchy type singular integral equation. The unknown function which is the derivative of crack surface displacement is numerically solved and used in the calculation of stress intensity factors. Limited finite element calculations are done for verification of the results which demonstrate the strong dependence of stress intensity factors on geometric and material parameters.

1. Introduction

Functionally graded materials (FGMs) are nontraditional engineering materials that are used especially in coating applications such as thermal barrier or wear resistant coatings (Erdogan, 1995). They are inhomogeneous materials whose properties vary in a specified manner. In order to gain a better understanding of the fracture of functionally graded materials, different researchers have solved many crack problems associated with FGMs within the last couple of decades. Employing various material modeling approaches and solution techniques, many useful results have been obtained for different crack configurations and loading conditions. There exist now, a vast literature on this subject. A relatively less studied subject within this vast literature pertains to the periodic cracking of functionally graded materials.

In this study one such problem, namely, plane strain mode one periodic cracking of a functionally graded semi-infinite plane is considered. Granted, the problem at hand is a highly idealized one and even more realistic problems (such as an FGM coating bonded to a half plane) have already been solved as will be discussed in the forthcoming literature survey. The distinguishing feature of this study, however, is that an analytical solution with a certain subtlety (to be discussed in Section 2) is being presented. In the earlier studies, either finite element or some other approximate method is used; or analytical solutions are obtained for some special materials which have variable thermal properties but constant elastic properties. Hence to the best of authors' knowledge, the analytical solution to the problem presented here, has not been published yet. The relevant literature is briefly reviewed in the following. The scope of the literature survey is restricted to periodic crack problems in linear elastic materials under thermal or mechanical loads.

Periodic cracks in homogeneous materials are investigated by many researchers. Earliest works belong to Benthem and Koiter (1973), and Bowie (1973) who solved the problem of a half-plane with periodic edge cracks by using different approaches. Nisitani et al. (1973) considered a row of internal cracks in a semi-infinite plane under uniform tension. Nemat-Nasser et al. (1978) addressed the issue of stability of crack growth under specific thermal stress conditions by considering a half plane containing two sets of interacting periodic edge cracks which are equally spaced but of unequal lengths. Stress intensity factors (SIF) used in the stability analysis are obtained from the solution of a singular integral equation with Cauchy type singularity. Isida (1979) also considered an array of parallel edge cracks in a semi-infinite plane under uniform tension. Some results from Nisitani et al. (1973) and Isida (1979) are given in Murakami (1987). Nied (1987) considered the elasticity problem of an infinite array of periodic internal cracks in a half plane, such that edge cracks could also be obtained as a special case. Dependence of SIFs and crack opening displacements on crack spacing has been investigated. In Nied (1987), instead of taking the usual approach of formulating the problem by using the derivative of the crack surface displacement (which leads to a singular integral equation

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