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**Engineering Analysis with Boundary Elements** 





## Truly meshless localized type techniques for the steady-state heat conduction problems for isotropic and functionally graded materials

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## ABSTRACT

A numerical solution of steady-state heat conduction problems is obtained using the strong form meshless point collocation (MPC) method. The approximation of the field variables is performed using the Moving Least Squares (MLS) and the local form of the multiquadrics Radial Basis Functions (LRBF). The accuracy and the efficiency of the MPC schemes (with MLS and LRBF approximations) are investigated through variation (i) of the nodal distribution type used, i.e. regular or irregular, ensuring the so-called positivity conditions, (ii) of the number of nodes in the total spatial domain (TD), and (iii) of the number of nodes in the total spatial domain (TD), and (iii) of the number of nodes in the support domain (SD). Numerical experiments are performed on representative case studies of increasing complexity, such as, (a) a regular geometry with a constant conductivity and uniformly distributed heat source, (b) a regular geometry with a spatially varying conductivity and non-uniformly distributed heat source, and (c) an irregular geometry in case of insulation of vapor transport tubes, as well. Steady-state boundary conditions of the Dirichlet-, Neumann-, or Robin-type are assumed. The results are compared with those calculated by the Finite Element Method with an in-house code, as well as with analytical solutions and other literature data. Thus, the accuracy and the efficiency of the method are demonstrated in all cases studied.

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## 1. Introduction

The subject of heat transfer is of fundamental importance in many branches of engineering and science. Furthermore, its study provides vital economical and efficient solutions for a plethora of critical problems. It refers to the section of engineering science that studies the energy transport between material bodies due to a temperature difference [1–4]. Additionally, modelling heat transport and temperature variations within biological tissues and body organs is an important issue in medical thermal therapeutic applications [5]. Heat transfer problems of modern technological value usually cannot be solved in an analytical manner, except for a few simplified cases. Thus, traditional numerical techniques, such as finite differences (FDM) [6,7], finite volume methods (FVM) [8], finite element methods (FEM) [9], and boundary element methods (BEM) [10], have been effectively and routinely applied.

In spite of their great success, the traditional numerical methods still have some elementary drawbacks that impair their

\* Corresponding author. E-mail addresses: Eugene.Skouras@iceht.forth.gr, Eugene@iceht.forth.gr (E.D. Skouras). computational efficiency and even limit their applicability to more practical problems, particularly in three-dimensional problems. The main reasons of deficiency are related to the use of low order piecewise polynomial approximations, and the necessity to create a mesh in the application domain and its boundary. As a result, the numerical solution depends strongly on the mesh properties. More precisely, the finite volume method (FVM) and the finite element method (FEM) have been the dominant numerical schemes for a variety of practical engineering and physical problems, since they have the advantage of being applicable at irregular geometries. Despite the fact that mesh generation can be fully automated in two dimensions, this can be a troublesome procedure at higher dimensions, usually demanding substantial human intervention. Thus, in the majority of heat transfer problems, mesh generation is a far more time consuming and expensive task than the solution of the partial differential equations (PDEs) itself.

Owing to the difficulty of the traditional numerical schemes in the mesh generation, new numerical methods, generally called "meshless" methods (also called "meshfree" methods), have been developed in recent years. Meshless methods emerged as a potential alternative for solutions in computational mechanics, and a variety of such approaches have appeared [11, 12 and references therein]. Meshless techniques overcome these

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