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Meshless local Petrov–Galerkin method for coupled thermoelasticity analysis of a functionally graded thick hollow cylinder

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ABSTRACT

In this article, coupled thermoelasticity (without energy dissipation) based on Green–Naghdi model is applied to functionally graded (FG) thick hollow cylinder. The meshless local Petrov–Galerkin method is developed to solve the boundary value problem. The Newmark finite difference method is used to treat the time dependence of the variables for transient problems. The FG cylinder is considered to be under axisymmetric and plane strain conditions and bounding surfaces of cylinder to be under thermal shock loading. The mechanical properties of FG cylinder are assumed to vary across thickness of cylinder in terms of volume fraction as nonlinear function. A weak formulation for the set of governing equations is transformed into local integral equations on local subdomains by using a Heaviside test function. Nodal points are regularly distributed along the radius of the cylinder and each node is surrounded by a uni-directional subdomain to which a local integral equation is applied. The Green–Naghdi coupled thermoelasticity equations are valid in each isotropic subdomain. The temperature and radial displacement distributions are obtained for some grading patterns of FGM at various time instants. The propagation of thermal and elastic waves is discussed in details. The presented method shows high capability and efficiency for coupled thermoelasticity problems.

1. Introduction

To calculate the thermal stresses in structures subjected to mechanical and thermal shock loading in uncoupled theory of thermoelasticity, the temperature distribution should be obtained using Fourier heat transfer equation. Then, the temperature distribution is substituted into the governing equations for mechanical quantities. The uncoupled theory of thermoelasticity does not match with the real physical behaviors in high strain rate and highly varying thermal boundary conditions. There are some models for coupled theories of thermoelasticity, which simulate the mutual dependency between temperature and displacements. Coupled problems of thermoelasticity take into account the time rate of the first invariant of strain tensor. The dependency between the temperature and displacement fields results in coupling of elasticity and energy equations. This situation occurs when the time rate of change of thermal boundary conditions is comparable with the structural disturbances. When characteristic times of structural and thermal disturbances are of comparable magnitudes, the equations of motion of a cylinder are coupled with the energy equation. Green and Naghdi presented a model in coupled thermoelasticity which is called GN theory of thermoelasticity [1]. Their model simulates the finite speed thermal wave propagation in structures, which are excited by high rate thermal loads [2]. In the recent years, the coupled thermoelasticity analysis has been developed for isotropic homogeneous and also new materials such as functionally graded materials (FGMs) using numerical and analytical methods. Melnik [3] solved the coupled thermoelasticity equation in stress-temperature fields using discrete approximations. He also studied the properties of method on mathematical models of coupled thermoelasticity. The coupled thermoelasticity equations based on Green–Naghdi models were solved and studied using locally transversal linearization (LTL) technique and the numerical inverse Laplace transform method by Taheri et al. [4]. They studied the propagation and reflection of thermal and mechanical waves in an annulus domain. Youssef [5] has analytically solved the thermoelastic problem for an elastic homogeneous medium with cylindrical cavity. He unified generalized coupled thermoelasticity equations based on Lord–Shulman (LS) and Green–Lindsay (GL) in one system and solved it for the problem. The unified formulation for generalized coupled thermoelasticity based on LS, GL and GN is proposed by Bagri and Eslami [6]. They solved the unified formulation for isotropic and homogeneous hollow spheres and cylinders using analytical method [6] and for functionally graded cylinders

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