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Shape variable radial basis function and its application in dual reciprocity boundary face method

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ABSTRACT

The radial basis functions (RBFs) is an efficient tool in multivariate approximation, but it usually suffers from an ill-conditioned interpolation matrix when interpolation points are very dense or irregularly spaced. The RBFs with variable shape parameters can usually improve the interpolation matrix condition number. In this paper a new shape parameter variation scheme is implemented. Comparison studies with the constant shaped RBF on convergence and stability are made. Results show that under the same accuracy level, the interpolation matrix condition number by our scheme grows much slower than that of the constant shaped RBF interpolation matrix with increase in the number of interpolation points. As an application example, the dual reciprocity method equipped with the new RBF is combined with the boundary face method to solve boundary value problems governed by Poisson equations. Numerical results further demonstrate the robustness and better stability of the new RBF.

1. Introduction

The boundary element method (BEM) is an efficient alternative numerical technique for solving partial differential equations (PDE). For problems governed by Laplace equation, Helmholtz equation and linear diffusion-reaction equation, it has been widely used. In this method, the PDE is converted to an equivalent boundary integral equation (BIE) using Green's theorem and a fundamental solution. Thus, its main advantage over the classic domain methods, such as finite element method (FEM) and finite difference method (FDM), is the need of boundary only discretization together with a high rate of convergence. Nevertheless, when dealing with inhomogeneous problems, non-linear problems and more general linear PDEs for which fundamental solutions are unavailable or inconvenient, the BEM becomes less attractive since it is inevitable to discretize the considered domain for calculating the domain integrals that remain in the BIEs for the above problems.

To avoid the domain integrals, an alternative method named dual reciprocity method (DRM) was proposed by Nardini and Brebbia [1]. In this approach, the inhomogeneous term of the PDE is approximated by a series of simple functions and transformed to the boundary integrals employing particular solutions of considered problem. Since the accuracy and the stability of the solution depend largely on that of the approximation, the choice of the approximating functions in the series is usually of crucial importance in the DRM formulation. The most widely used approximating functions in DRM are radial basis functions (RBFs).

The RBF interpolation was pioneered by Hardy. After that there is a wide range of applications of the RBF, especially in meshless methods for solving partial differential equations [2,3]. Despite its simplicity and efficiency, the RBF interpolation suffers from a contradiction between accuracy and stability, which can be expressed in a form similar to the uncertainty principle in quantum mechanics [4]. The RBF interpolation matrix condition number becomes very large when the interpolation points are dense or irregularly spaced, and the ill-conditioned interpolation matrix limits its further application especially in large scale problems. To guarantee the robust of the interpolation many researchers have sought for the theoretical results about the convergence and stability of the RBF interpolation [4–8]. So far many methods have been proposed, such as compactly supported RBF, multilevel method, precondition method, domain decomposition method, truncated RBF method, RBF with variable shape parameter and knot optimization method [9]. The present paper focuses on the RBF with variable shape parameter.

The concept of variable shape parameters in the RBF interpolation has been proposed by many researchers, e.g. Kansa and Carlson [10]. The main idea is to determine the shape parameter of a RBF in terms of the local density of its corresponding interpolation point. Thus the columns of the interpolation matrix elements are more distinct, and the condition number becomes smaller. However, new problems may be caused by the shape parameter variation such as a singular interpolation matrix, lower convergence rate and difficulties to choose the variation schemes [11]. Many previous researches

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