Regularized solutions with a singular point for the inverse biharmonic boundary value problem by the method of fundamental solutions

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1. Introduction

Inverse problems have recently attracted attention in science and engineering. The Cauchy problems of elliptic partial differential equations are well-known inverse problems. Many researchers have proposed numerical methods for the Cauchy problem of the Laplace equation by applying various numerical methods for solving partial differential equations: for example, the finite difference method (FDM), the finite element method (FEM), and the boundary element method (BEM) [8,19,23–27]. The conventional methods, such as the FDM or the spectral collocation method in multiple-precision arithmetic, cannot successfully solve the problem whose solution has a singular point outside the computational domain (see [9,24] for example).

The method of fundamental solutions (MFS) is effective for easily and rapidly solving elliptic well-posed direct problems in complicated domains. Mathon and Johnston [17] first showed numerical results obtained by the MFS. The papers [1,11] discuss some mathematical theories on the MFS. Both of the BEM and the MFS are well-known boundary methods, which discretize original problems by using the fundamental solutions. The MFS does not require any treatments for the singularity of the fundamental solution, while the BEM requires singular integrals. The MFS is a true meshless method, and can easily be extended to higher dimensional cases.

Hon and Wei [6] first applied the MFS to solve the one-dimensional inverse heat conduction problem. The MFS was also used to solve the Cauchy problems of the Laplace [14,28,30], the Helmholtz [10,13,15], and the biharmonic [16] equations. In this paper, as Wei et al. [27] applied the MFS to the Cauchy problems of second order elliptic equations, we use the MFS to directly discretize the Cauchy problem of the biharmonic equation. The biharmonic equation is an elliptic partial differential equation with fourth order, appearing in areas of linear elasticity and Stokes flows. This equation and the Laplace equation are important basic equations in mathematics and physics. Numerical methods for solving boundary value problems of these equations have already been established. However, the Cauchy problem of the biharmonic equation is an ill-posed problem, where the solution has no continuous dependence on the boundary data. In other words, a small noise contained in the given Cauchy data may affect sensitively on the accuracy of the solution. We notice that the noisy Cauchy data are not biharmonic. In a related matter, Drombosky et al. [2] examined the relationship between the accuracy of the MFS for the Laplace equation and the effective condition number when the boundary data are not harmonic.

Our problem is discretized directly by the MFS. As another method, our problem is decomposed into two Cauchy problems of...