Two-dimensional version of Sternberg and Al-Khozaie fundamental solution for viscoelastic analysis using the boundary element method

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ABSTRACT

In this work a general and concise two-dimensional fundamental solution is obtained for quasi-static linear viscoelastic problems using the boundary element method. For this purpose, the three-dimensional fundamental displacement, derived by Sternberg and Al-Khozaie from the generalization of Navier equation, is integrated with respect to \( z \)-coordinate. A time formulation is constructed from the viscoelastic Reciprocity Principle, defined in terms of the Stieltjes integral and the material functions are acquired by means of Boltzmann’s rheological model. The collocation method and a semi-analytical procedure for the singular boundary integral are employed to the numerical analysis of the boundary integral. The Gaussian quadrature, the analytical method and an incremental approach are used to deal with the convolution integral. As the latter has presented the best performance, it is employed in most analyses of the examples. Finally, numerical results of problems, found in the literature, are presented in order to validate the formulation and the two-dimensional fundamental solution.

1. Introduction

At the end of the 19th century, after relevant contribution of Boltzmann to the viscoelasticity [1], the theory was only improved from the emergence of polymeric materials. Since then, it has been applied to several materials employed in modern engineering, in which elastic and viscous features are evidenced.

The arising of computational tools and numerical methods turned possible a quick and accurate evaluation of mechanical behaviours of bodies presented in more complex problems. The boundary element method has become an interesting method for applications in viscoelastic problems, in spite of the inconvenience of finding a fundamental viscoelastic solution of easy manipulation. For that, alternative procedure to obtain the solution of the Navier equation for a concentrated load [2] is adopted by almost all researchers and consists in the use of the elastic-viscoelastic Correspondence Principle [3], which involves Laplace transforms of elastic solutions and rheological models representative of the behaviour [4–8].

Next some relevant works are cited in a brief review of linear viscoelasticity. In the Rizzo and Shippy’s work [9], the final results obtained from a viscoelastic integral equation in Laplace domain are transformed into the time domain by means of a numerical inversion procedure. A time formulation is presented by Syngellakis [4] and Syngellakis and Wu [5,10] for a quasi-static and dynamic analyses and fracture problems in polymers, with fundamental solution obtained from the Correspondence Principle. Schanz [11] and Schanz et al. [12,13] describe the same analyses in time domain using the Lubich quadrature [14,15] to deal with an integral convolutions, in which the weighting function is the fundamental solution in the Laplace domain.

Dynamic analysis is also treated by Pérez-Gravílán and Alíabadi [16] employing the symmetric Galerkin boundary element method for frequency domain. Ashrafi and Farid [17] derived an integral formulation applying Kelvin’s fundamental solution with material constants expressed as time-dependent functions and a differential constitutive equation of a standard linear solid (SLS) viscoelastic model to the polymer analysis. Although domain integrals arise in their formulation due to viscous effects, only the boundary of the problem is discretized.

Mesquita and Coda [18] also present the integral formulation for the Kelvin and Boltzmann models considering a differential constitutive relation. In this work, internal elements are not used on the problem. In two recent works [19,20], a two-dimensional viscoelastic analysis is presented, using internal cells only in the first one and a differential constitutive relation for Kelvin and Boltzmann models, implemented in boundary element atmosphere.

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