A nonlinear complementarity approach for elastoplastic problems by BEM without internal cells

Qin Deng*, ChunGuang Li, ShuiLin Wang, Hong Zheng, XiuRun Ge

State Key Laboratory of Geomechanics and Geotechnical Engineering, Institute of Rock and Soil Mechanics, Chinese Academy of Sciences, Wuhan 430071, China

A R T I C L E   I N F O

Article history:
Received 16 August 2010
Accepted 1 October 2010
Available online 28 October 2010

Keywords:
Complementarity approach
Elastoplastic problems
Boundary element method
Domain integrals
Radial integration method

A B S T R A C T

A nonlinear complementarity approach is presented to solve elastoplastic problems by the boundary element method, in which the equations are formulated by stress equations and complementarity function obtained from the plasticity constitutive law. The domain integrals involved are transformed into boundary integrals by radial integration method, using compactly supported radial basis functions. Two numerical examples demonstrate the algorithm’s applicability and effectiveness.

© 2010 Elsevier Ltd. All rights reserved.

1. Introduction

Boundary element method (BEM) has been established as one of the major numerical techniques for solving partial differential equations. Because of the reduction of the dimension of the problem by one, accuracy and low computational complexity, BEM has been successfully applied to some nonlinear problems, for example, elastoplastic problems, heat diffusion and contact problems, etc. [1].

An important task in nonlinear BEM is to solve the system of nonlinear equations. Explicit algorithms described in detail by Telles [2] and Banerjee [1] were the earliest methods, which can be roughly classified into two groups, i.e., the initial stress methods and the initial strain methods. In this technique, the boundary unknowns are the primary variables of the equations, and hence the amount of computing is modest. However, solution convergence is slow and even may not be achieved for some complicated cases [3]. Later, implicit methods [4–6] were introduced. The equations involved are developed in terms of stress or strain increments and can be solved by various iterative algorithms [7]. Among these works, Bonnet and Mukherjee [5] applied the consistent tangent operator method to BEM. The consistent tangent operator method was firstly proposed by Simo and Taylor [8] in the context of FEM and is easy to code. The variable stiffness technique proposed by Banerjee et al. [9] constructs the equations in the incremental boundary unknowns. If small increments are used, no iteration is needed. Recently, Gao [10] proposes a new incremental variable stiffness iterative algorithm, in which only the plastic flow factors are used as the primary variables. Accordingly, the time and memory consumed are substantially reduced.

The presence of domain integrals, which detracts from the elegance of the formulation of BEM, is a challenge for BEM. During the past decades, substantial effort has been expended to avoid, minimize or eliminate the domain discretization [11–26]. The following four methods are widely used. The first approach is the partial solution method [17], which splits the density function into a particular solution and the solution of the associated homogeneous equation. However, it needs to describe the behavior of the initial stress or strain in the analytic form, and hence it may be difficult to do in some cases [19]. The second approach is the dual reciprocity boundary element method [18–19]. This method approximates domain variables (body forces, initial states, nonlinear terms, etc.) using a series of prescribed basis function, and transforms the domain integrals into boundary integrals by integrating by part. The third approach is the multiple reciprocity boundary element method [20–22]. In this approach, higher order fundamental solutions are used. Moreover it is necessary to determine along the boundary not only the domain variables, but also a series of its Laplacian. Recently, much attention has been paid to the radial integration method presented by Gao [23], which can transform any domain integral into a boundary integral and a radial integral. Now it has been successfully employed to solve thermoelasticity problems [24], elastic inclusion problems [25] and heat conduction problems [26]. In what follows, the radial integration method will be described in detail.

In the present work, the complementary theory is introduced into elastoplastic analysis by BEM, both stress components and plastic flow factors are considered as primary unknowns, and the domain integrals involved are transformed into boundary integrals by the radial integration method using compactly supported radial basis functions.