A novel boundary element approach for solving the anisotropic potential problems

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Abstract

This presentation is mainly devoted to the research on the regularization of indirect boundary integral equations (IBEs) for anisotropic potential problems. Based on a new idea, a novel regularization technique is pursued, in which the regularized IBEs excluding the CPV and HFP integrals are established. The proposed method has many advantages. First, it does not need to calculate multiple integral as the Galerkin method, so it is simple and easy for programming. Second, it can compute boundary quantities directly without transforming them into isotropic ones so that no inverse transform is required. Finally, the gradient IBES are independent of the potential IBEs and they can provide variously useful equations. Numerical examples show that a better precision and high computational efficiency can be achieved by the present method.

1. Introduction

Anisotropic media always occur in nature, such as woods, crystals and sedimentary rocks, and can also be produced artificially, such as laminated and fiber-reinforced construction and electronic materials, cables, cylinders and tubes. Increase in the use of these materials in structural applications has considerably renewed the interest in the solutions to potential problems in anisotropy. Generally, the anisotropic potential problems include the problem of heat conduction in anisotropic media, the problem of subsurface flow in anisotropic media, and the problem of torsion in anisotropic uniform bar.

It is well known that the numerical analysis for anisotropic problems has been performed by utilizing experimental, analytical and numerical methods. The usual numerical method such as the finite difference method (FDM), finite element method (FEM), boundary element method (BEM), and meshfree method can be applied to solve such problem. The FEM has long been a dominant numerical technique in the simulation of many industrial problems. However, this method requires discretizing the whole computational domain, which is often computational costly and sometimes mathematically troublesome for some complex problems. For solving the infinite domain problems, the FEM needs to truncate infinite domain into an artificial finite region with subtle artificial boundary conditions or absorbing layers. This truncation can be somewhat arbitrary largely based on various trial-error or empirical approaches. As a domain discretization technique, the FEM is also less effective for inverse problems in which measurement is often only accessible on the boundary. As an alternative approach, the BEM has been well known to avoid such drawbacks [1–4]. It is well known that the BEM can reduce the discretization complexity by one dimension compared to that of the FEM. It is also worth noting that the derivatives of the physical quantity can be calculated directly from the original boundary integral equations, so that the solution accuracy of both the physical quantities and its derivatives has the same orders of magnitude. In sharp contrast, the domain-type numerical methods, such as the FEM, do not have such good property.

As the price pays for such merits, the standard BEM formulation, however, has to evaluate varied orders of singular integrals, which requires great care and significant analysis. In the past decades, tremendous effort was devoted to derive convenient integral forms or sophisticated computational techniques for calculating the troublesome singular integrals. These proposed methods can be summarized on the whole as two categories: the local and the global strategies. The local strategies are employed to calculate the singular integrals directly. They usually include,