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Numerical evaluation of arbitrary singular domain integrals based on radial integration method

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ABSTRACT

In this paper, a new approach is presented for the numerical evaluation of arbitrary singular domain integrals based on the radial integration method. The transformation from domain integrals to boundary integrals and the analytical elimination of singularities can be accomplished by expressing the non-singular part of the integration kernels as polynomials of the distance r and using the intrinsic features of the radial integral. In the proposed method, singularities involved in the domain integrals are explicitly transformed to the boundary integrals, so no singularities exist at internal points. Some numerical examples are provided to verify the correctness and robustness of the presented method.

1. Introduction

Since the work of Jaswon [1] in potential problems and Rizzo [2] in elastostatics, the boundary element method (BEM), with the distinct feature that only the boundary of the problem needs to be discretized into elements, has been developed rapidly to such a level that it can be applied to solve very complicated engineering problems [3,4]. However, in the presence of body forces, the resultant integral equations include domain integrals [5,6]. In order to evaluate these integrals, the domain of the problem needs to be discretized into internal cells. Although the cell-integration scheme can give accurate results [7], the discretization of the domain into cells eliminates, to a certain extent, the advantage of BEM in that only the boundary of the problem needs to be discretized into elements.

During the past three decades, various techniques have been developed to overcome this deficiency when evaluating domain integrals (see works of Becker [8] and Kane [9] for detailed reviews). The commonly used technique is to apply some special schemes to transform domain integrals appearing in conventional boundary integral equations into boundary integrals, thus avoiding the discretization of the internal domain. The most popular technique is the dual reciprocity method (DRM) proposed by Nardini and Brebbia in 1982 [10]. The method, approximating the body force effect quantities by a series of prescribed basis functions, transforms the domain integrals to the boundary by employing particular solutions that are derived from the differential operator of the problem based on these basis functions. Since the publication of the first book on the DRM by Partridge et al. [11], this method has been extensively used by means of exploiting the radial basis functions (RBF) [12–15]. As an extension of the idea of DRM, Nowak and Brebbia [16] developed a powerful technique called the multiple reciprocity method (MRM) to solve Poisson and Helmholtz equations. In MRM, the reciprocity theorem is repeatedly applied using a sequence of higher order fundamental solutions of Laplace operator to transformation the domain integrals to the boundary.

More recently, Gao [17,18] presented a new and simple technique, called the radial integration method (RIM), for transforming domain integrals into boundary integrals based on a radial integration technique. Since this method is based on pure mathematical treatments, it can be applied to transform any type of domain integrals into boundary integrals without using the Laplace operator and particular solutions of the problem. For high-order singular domain integrals (i.e., strongly, hyper and super singular domain integrals), the technique presented by Gao [18] is only suitable for evaluating domain integrals in which the non-singular part of kernel functions satisfies the annular function condition and, therefore, cannot be applied to evaluate high-order singular domain integrals including general functions of non-singular parts.

In this paper, a new and simple method is presented for the evaluation of arbitrarily high-order singular domain integrals in a unified way based on the radial integration method. The radial integral is evaluated analytically by expressing the non-singular part of the integration kernel as polynomials of the global distance r and the singularities are removed analytically through transforming the domain integrals to the boundary. The approach

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