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## The multi-domain hybrid boundary node method for 3D elasticity

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#### ABSTRACT

In this paper, a multi-domain technique for 3D elasticity problems is derived from the hybrid boundary node method (Hybrid BNM). The Hybrid BNM is based on the modified variational principle and the Moving Least Squares (MLS) approximation. It does not require a boundary element mesh, neither for the purpose of interpolation of the solution variables nor for the integration of energy. This method can reduce the human-labor costs of meshing, especially for complex construction. This paper presents a further development of the Hybrid BNM for multi-domain analysis in 3D elasticity. Using the equilibrium and continuity conditions on the interfaces, the final algebraic equation is obtained by assembling the algebraic equation for each single sub-domain. The proposed multi-domain technique is capable to deal with interface and multi-medium problems and results in a block sparsity of the coefficient matrix. Numerical examples demonstrate the accuracy of the proposed multi-domain technique.

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### 1. Introduction

Multi-domain (multi-zone) formulations play an important part in numerical analysis when dealing with problems involving interface or dissimilar materials, such as composite materials, etc. Besides, in the case of problems with complicated boundary or a large number of degrees of freedom (DOFs), the use of multidomain techniques results in a block sparsity matrix and a better computational efficiency may be obtained. The multi-domain techniques for potential and elasticity problems have been widely investigated in boundary element method (BEM) and some other numerical methods. The basic idea of the multi-domain techniques in BEM is that the whole domain is broken up into separate sub-domains, and a boundary integral equation is obtained for each sub-domain. By assembling all contributions of boundary integral equations for each sub-domain, the final system equation can be formed. Ramsak and Skerget [1] present an efficient 3D multi-domain boundary element method for solving problems governed by the Laplace equation. A symmetric Galerkin multizone boundary element formulation for elasticity problems is proposed by Layton et al. [2]. Gray and Paulino [3] present a symmetric Galerkin boundary integral method for interface and multi-zone problems. Gao and Davies [4] propose a new technique for 3D elasticity BEM with corners and edges. Gao and Yang [5] derive a boundary integral equation for multi-medium elasticity problems from the boundary-domain integral equation for single medium elasticity problems. A multi-domain boundary contour method (BCM) for interface and dissimilar material problems in elasticity problems is proposed by Phan and Mukherjee [6]. The multi-domain techniques in all of the above-mentioned works are based on the continuity and equilibrium conditions across the interface, namely the continuity of potential (Laplace) or displacement (elasticity) and the equilibrium of flux (Laplace) or traction (elasticity).

In the conventional BEM, the task of mesh generation for complex geometries is often time-consuming and prone to errors even though it only needs discretization of the boundary, and it has difficulties with remeshing in problems involving moving boundaries, large deformations or crack propagation. In the last decades, a new class of numerical methods, namely, the meshless or meshfree methods were developed. One of the main purposes of the meshless techniques is to reduce the human-labor costs required for meshing the domains of complex-shape. There are many meshless techniques have been proposed so far, including the element free Galerkin method (EFG) [7], the meshless local Petrov-Galerkin (MLPG) approach [8], the boundary node method (BNM) [9] and the hybrid boundary node method (Hybrid BNM), etc. The Hybrid BNM is proposed by Zhang et al. [10-14] for potential and elasticity problems and has been developed by Miao et al. [15,16], which combines the MLS approximation [17] scheme with the hybrid displacement variational formula. It not only reduce the spatial dimensions by one like BEM or BNM, but also does not require a boundary element mesh, neither for the purpose of interpolation of the solution variables nor for the integration of energy. In fact, the Hybrid BNM requires only discrete node located on the surface of the domain and its

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