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## Falkner-Skan problem for a static or moving wedge in nanofluids

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### ABSTRACT

The steady two-dimensional boundary layer flow past a static or a moving wedge immersed in nanofluids is investigated numerically. An implicit finite difference scheme known as the Keller-box method and the NAG routine DO2HAF are used to obtain the numerical solutions. Three different types of nanoparticles, namely copper Cu, alumina Al<sub>2</sub>O<sub>3</sub> and titania TiO<sub>2</sub> with water as the base fluid are considered. The effects of the governing parameters on the fluid flow and heat transfer characteristics are analyzed and discussed. It is found that Cu-water has the highest skin friction coefficient and the heat transfer rate at the surface compared with the others. The effect of the solid volume fraction of nanoparticles on the fluid flow and heat transfer characteristics is found to be more pronounced compared to the type of the nanoparticles.

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### 1. Introduction

Nanofluids are a new class of nanotechnology-based heat transfer fluids engineered by dispersing nanometer-sized particles with typical length scales on the order of 1 to 100 nm in traditional heat transfer fluids (Das et al. [1]). Nanofluids are produced by dispersing the nanometer-scale solid particles into base liquids with low thermal conductivity such as water, ethylene glycol (EG), oils, etc. (Wang and Mujumdar [2]). The nanoparticles can be made of metal, metal oxide, carbide, nitride and even immiscible nanoscale liquid droplets (Chen and Ding [3]). The shape of nanoparticles can be spherical, rod-like or tubular shapes and can be dispersed individually. Nanofluids can be categorized as Newtonian or non-Newtonian fluids. The term of nanofluids was first introduced by Choi [4] to describe this new class of fluid. There are mainly two techniques used to produce nanofluids which are the single-step and the two-step method (see Akoh et al. [5] and Eastman et al. [6]). Both of these methods have advantages and disadvantages as discussed by Wang and Mujumdar [2]. The materials with sizes of nanometers possess unique physical and chemical properties (Das et al. [1]). They can flow smoothly through microchannels without clogging them because it is small enough to behave similar to liquid molecules (Kanafer et al. [7]). This fact has attracted many researchers such as Abu-Nada [8], Tiwari and Das [9], Maïga et al. [10], Polidari et al. [11], Oztop and Abu-Nada [12], Nield and Kuznetsov [13], Kuznetsov and Nield [14] and Muthtamilselvan et al. [15] to investigate the heat transfer characteristics in nanofluids, and they found that in the presence of the nanoparticles in the fluids increases appreciably the effective thermal conductivity of the fluid and consequently enhances the heat transfer characteristics. There have been many studies in the literature to better understand the mechanism behind the enhanced heat transfer characteristics. An excellent collection of papers on this topic can be found in the book by Das et al. [1] and in the review papers by Buongiorno [16], Daungthongsuk and Wongwises [17], Trisaksri and Wongwises [18], Wang and Mujumdar [2,19,20], and Kakaç and Pramuanjaroenkij [21].

The boundary layer flow over a static or a moving wedge in a viscous fluid (regular fluid) has been considered by Riley and Weidman [22] and Ishak et al. [23], which is an extension of the flow over a static wedge considered by Falkner and Skan [24]. They employed a similarity transformation that reduces the partial differential boundary layer equations to a nonlinear third-order ordinary differential equation before solving it numerically. A large amount of literatures on this problem has been cited in the books by Schlichting and Gersten [25] and Leal [26] as well as in the paper by Ishak et al. [23]. To the best of our knowledge, there are a few studies on the Falkner–Skan problem that considered various values of

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