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Haar wavelet collocation method for the numerical solution of boundary layer fluid flow problems

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ABSTRACT

Based on Haar wavelets an efficient numerical method is proposed for the numerical solution of system of coupled Ordinary Differential Equations (ODEs) related to the natural convection boundary layer fluid flow problems with high Prandtl number (Pr). The numerical study of these flow models is necessary as the existing literature is more focused on the flow problems with small values of Pr. In this work, the problem of natural convection which consists of coupled nonlinear ODEs is solved simultaneously. The ODEs are obtained from the Navier Stokes equations through the similarity transformations. The effects of variation of Pr on heat transfer are investigated. Performance of the Haar Wavelets Collocation Method (HWCM) is compared with the finite difference method (FDM), Runge–Kutta Method (RKM), homotopy analysis method (HAM) and exact solution for the last problem. More accurate solutions are obtained by wavelets decomposition in the form of a multi-resolution analysis of the function which represents solution of the given problems. Through this analysis the solution is found on the coarse grid points and then refined towards higher accuracy by increasing the level of the Haar wavelets. Neumann's boundary conditions which are problematic for most of the numerical methods are automatically coped with. A distinctive feature of the proposed method is its simple applicability for a variety of boundary conditions. Efficiency analysis of HWCM versus RKM is performed using Timing command in Mathematica software. A brief convergence analysis of the proposed method is given. Numerical tests are performed to test the applicability, efficiency and accuracy of the method.

1. Introduction

High Prandtl number fluids are frequently encountered in industry such as fluids used as a heat sink in electrical transformer with $Pr = 47100$ at 273 K, hydrocarbon polymers or silicons used in some chemical processes and substances such as glycerine with $Pr = 8470$ at 273 K. Geological flows involve fluids with very large Pr ($Pr = 1000$ for magmas and at least 10^{23} for the earths mantle), engine oil ($Pr = 100$–$40{,}000$), starting plumes rising through viscous oils ($Pr = 10^4$), etc [21,35]. The governing partial differential equations in cartesian co-ordinates which consist of continuity, momentum and energy for these flow models are converted to ODEs through similarity transformations. Due to scarcity of efficient numerical methods, the resulting systems of simultaneous coupled nonlinear ODEs are often very challenging for the existing methods in the case of large value of Pr. Efficient numerical methods are needed for numerical solution of highly nonlinear system of ODEs where the analytical solutions appear infeasible. To the best of our knowledge only a few papers are dedicated to the numerical solution of these types of ODEs. In [6,41] the authors discussed the existence and uniqueness of solution to the second-order systems. The numerical methods include FDM and adjoint operator methods [30], reproducing kernel space [10], variational iteration method (VIM) [28], third-degree B-spline [3], sinc-collocation method [7], Chebychev finite difference method [36]. Among the numerical methods, RKM with adaptive or uniform step sizes coupled with nonlinear shooting method [30] is mostly used for the numerical solution of the coupled system of ODEs arising from the nonlinear fluid models through the similarity transformations. This is done in two stages:

(i) Shooting method is used to integrate the boundary value problems as an initial value problem with guesses for the unknown initial values,
(ii) Initial value problem is reduced to a system of first-order ODEs and then solved by RKM.

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