Sequential Lagrangian-MILP approaches for Unit Commitment problems

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\begin{abstract}
The short-term Unit Commitment (UC) problem in hydro-thermal power generation is a fundamental problem in short-term electrical generation scheduling. Historically, Lagrangian techniques have been used to tackle this large-scale, difficult Mixed-Integer NonLinear Program (MINLP); this requires being able to efficiently solve the Lagrangian subproblems, which has only recently become possible (efficiently enough) for units subject to significant ramp constraints. In the last years, alternative approaches have been devised where the nonlinearities in the problem are approximated by means of piecewise-linear functions, so that UC can be approximated by a Mixed-Integer Linear Program (MILP); in particular, using a recently developed class of valid inequalities for the problem, called “Perspective Cuts”, significant improvements have been obtained in the efficiency and effectiveness of the solution algorithms. These two different approaches have complementary strengths; Lagrangian ones provide very good lower bounds quickly, but they require sophisticated heuristics—which may need to be changed every time that the mathematical model changes—for producing actual feasible solutions. MILP approaches have been shown to be able to provide very good feasible solutions quickly, but their lower bound is significantly worse. We present a sequential approach which combines the two methods, trying to exploit each one’s strengths; we show, by means of extensive computational experiments on realistic instances, that the sequential approach may exhibit significantly better efficiency than either of the two basic ones, depending on the degree of accuracy requested to the feasible solutions.
\end{abstract}

\section{Introduction}

The short-term Unit Commitment (UC) problem requires to exploit the different technical characteristics of a set of generating units—primarily, hydro valleys and thermal power generation systems—in order to satisfy a forecasted energy demand, over a given time horizon (typically one day or one week), at minimum cost. Each unit is subject to specific operating constraints which must be taken into account in order to produce a feasible commitment. Thermal units typically are subject to upper and lower bounds over the produced power when the unit is operational, minimum up- and down-time constraints, and ramp constraints. Hydro units are subject to bounds on the discharge rate, spillage limits and reservoir storage; furthermore, for cascaded units the effect on downstream reservoirs has to be taken into account. The version of UC we consider is the one where all units are under the control of one entity, a typical situation of the era of monopolistic producers; however, this problem is still very relevant in today’s deregulated electricity markets [34,40], although other issues, such as electricity price forecast [1], need now to be taken into account. This is so both because UC still need to be solved at various stages of the decision chain of electricity producers, consumers and regulating bodies, and because the algorithmic approaches developed for the UC can often be extended to restructured electric power markets [3,12,34].

Several variants of UC have been investigated during the last 30 years, and several specialized algorithmic approaches have been proposed to solve, possibly approximately, this problem; in fact, being UC a large-scale Mixed-Integer NonLinear Program (MINLP), it is rather difficult to solve instances of realistic size within the time limits required by operational environments. Among the most efficient specialized algorithmic approaches for (UC), Lagrangian Relaxation (LR) methods [5,7,10,11,20,35,36,39,45] surely play a major role. These approaches exploit the \textit{spatial structure} of the problem, that is, the fact that by removing the constraints that tie the different units together one obtains a set of disjoint Single-Unit Commitment (1UC) problems, requiring to optimally operate one single (hydro or thermal) unit over the time horizon. Thus, the applicability of LR methods critically depend on being able to optimally solve the 1UC problems efficiently; in turn, this depends on the specific details of the operational constraints of the generating units that are represented in the mathematical...