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# Some properties of nonnegative integral majorization

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**Abstract**

A majorization permutahedron $M(a)$ is polytope defined by $M(a) = \{x \in \mathbb{R}^n : x \preceq a\}$. In this paper we look more precisely to $M_I^+(a)$, all positive integer vectors that are majorized by $a$, and we discuss about its cardinality.

**Keywords:** integral vector, majorization, permutahedron
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## 1 Introduction

Inequalities in matrix theory and specially majorization is one of the interesting areas that has been researched on it in several ways.

For a vector $x \in \mathbb{R}^n$ we say $x = (x_1, \dots, x_n) \in \mathbb{R}^n$ is nonincreasing if

$$x_1 \geq x_2 \geq ... \geq x_n.$$

For a vector $x = (x_1, \dots, x_n) \in \mathbb{R}^n$ we use the notation $x^\downarrow = (x_1^\downarrow, x_2^\downarrow, \dots, x_n^\downarrow)$ for the nonincreasing vector consisting elements of $x$.

Let $x = (x_1, \dots, x_n), y = (y_1, \dots, y_n) \in \mathbb{R}^n$. We say $x$ is majorized by $y$, $x \prec y$, if

$$\sum_{i=1}^{k} x_i^\downarrow \leq \sum_{i=1}^{k} y_i^\downarrow, \quad 1 \leq k \leq n$$

with the equality when $k = n$[3].

A majorization permutahedron is defined by $M(a) = \{x \in \mathbb{R}^n : x \preceq a\}$. Actually this is a special polytope associated with a majorization in $\mathbb{R}^n$, the set of all vectors majorized by $a$. . In [1] there are some works on the properties of majorization permutahedrons and their cardinality. In this paper we are interested in the set of nonnegative integer vectors majorized by $a$ and discussing about its cardinality. We use the notation $\mathbb{R}^n_+$ for the set of all nonnegative vectors in $\mathbb{R}^n$.

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