46^{th} Annual Iranian Mathematics Conference
25-28 August 2015
Yazd University

Containment problem for the ideal of fatted almost collinear closed points in $\mathbb{P}^2$

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Abstract

In this paper, we study the containment problem for the ideal of a zero dimensional closed subscheme $Z = cp_0 + p_1 + \cdots + p_n$ of $\mathbb{P}^2$, where all points p_i except p_0, lie on a line and p_0 is considered with multiplicity c. We determine some numerical invariants of the ideal of this type of configuration, that is, the least degree of the generators of $I(Z)^{(r)}$, the resurgence of $I(Z)$ as well as the Waldschmit's constant of $I(Z)$.

Keywords: Symbolic power, Resurgence, Fat point
Mathematics Subject Classification [2010]: 14N20, 13F20

1 Introduction

Let $R = \mathbb{K}[\mathbb{P}^N] = \mathbb{K}[x_0, x_1, \ldots, x_N]$ be the homogeneous coordinate ring of the projective space $\mathbb{P}^N$, where $\mathbb{K}$ is an algebraically closed field of arbitrary characteristic. Let I be a nontrivial homogeneous ideal of R. The r^{th} symbolic power of I is defined to be the ideal

$$I^{(r)} = \cap_{P \in \mathrm{Ass}(I)}(R \cap I^r R_P).$$

Equivalently, $I^{(r)}$ is the contraction of the ideal $I^r R_U$ to R, i.e.,

$$I^{(r)} = R \cap I^r R_U,$$

where U is the multiplicative closed set $R - \cup_{P \in \mathrm{Ass}(I)} P$.
A natural algebraic operation for investigating the algebraic structure of I is to study the behavior of its ordinary power I^r, for each positive integer r, i.e., the ideal generated by products of r elements of I. On the other hand, I^r determines a closed subscheme of $\mathbb{P}^N$, a geometric object that is defined by the intersection of those primary components of I^r which their radical are strictly contained in $\langle x_0, x_1, \ldots, x_N \rangle$, denoted by $I^{(r)}$. But contrary to I^r, the generators of $I^{(r)}$ can not be obtained easily. A natural way to obtain information about the generators of $I^{(r)}$, is to compare its generators with the generators of different ordinary powers of I. In this direction, it can be easily proved that $I^m \subseteq I^{(r)}$

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