Testing Statistical Hypothesis of exponential populations with multiply sequential order statistics

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Abstract
Sequential order statistics (SOS) coming from non-homogeneous exponential distributions are considered in this paper. The generalized likelihood ratio (GLRT) and the Bayesian tests are derived for testing homogeneity of the exponential populations. It is shown that the GLRT in this case is also scale invariant. The maximum likelihood and the Bayesian estimates of parameters are derived on the basis of observed SOS samples. Explicit expression for SOS-based Bayes factor (BF) are derived.

Keywords: Bayes, GLRT, Sequential order statistics, Estimation

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1 Introduction
Let $X_1, \cdots, X_n$ be independent and identically distributed (i.i.d.) random variables with a common distribution function (DF), say $F$, and denoted by $X_1, \cdots, X_n \overset{i.i.d.}{\sim} F$. Denote in magnitude order of $X_1, \cdots, X_n$ by $X_{1:n} \leq \cdots \leq X_{n:n}$, which are called order statistics (OSs). In engineering system reliability analyses, lifetimes of $r$-out-of-$n$ systems, say $T$, coincide to $X_{r:n}$ in which $X_1, \cdots, X_n$ stand for component lifetimes. When the component lifetimes $X_1, \cdots, X_n \overset{i.i.d.}{\sim} F$, the OSs are used for describing the system lifetime. Notice that failing a component does not change here the lifetimes of the surviving components. Motivated by Cramer and Kamps [1], the failure of a component may result in a higher load on the surviving components and hence causes the lifetime distributions change. In these cases, the system lifetimes may be adequate to model by the concept of sequential order statistics (SOSs) as an extension of OSs. Cramer and Kamps [1] considered the problem of estimating the parameters on the basis of $s$ independent SOSs samples under a conditional proportional hazard rates (CPHR) model, defined by $F_j(t) = F_0^{\alpha_j}(t)$ for $j = 1, \cdots, r$, where the underlying CDF $F_0(t)$ is the exponential distribution, i.e.

$$F_0(x; \sigma) = 1 - \exp \left\{ - \left( \frac{x}{\sigma} \right) \right\}, \quad x > 0, \quad \sigma > 0.$$  \hspace{1cm} (1)

This paper develops testing Statistical Hypothesis for homogeneity of the exponential populations in section 2. In section 3, the Bayesian approach is used and Bayes factor is derived for evaluating support of data for homogeneity of populations.

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