Fixed point theorems in probabilistic metric space and intuitionistic probabilistic metric space

Fatemeh mohmedi*
University of Shahrekord

Behnoosh Salimiyan
University of Shahrekord

Maryam Shams
Shahrekord University

Abstract

In this paper, we introduce the non-Archimedean Menger PM-space, Φ-functions, intuitionistic probabilistic metric space and then prove fixed point theorems for family of self-mapping and generalized contraction mapping.

Keywords: non-Archimedean probabilistic Menger space, intuitionistic probabilistic metric space, t-representable
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1 Introduction

The triangular norm (t-norm) and the triangular conorm (t-conorm) originated from the studies of probabilistic metric spaces [5, 6] in which triangular inequalities were extended using the theory of t-norm and t-conorm. Non-Archimedean probabilistic metric spaces first studied by Isratescu and Crivat [3]. Some fixed point theorems for mappings on non-Archimedean Menger spaces have been proved by Isratescu [1, 2]. Menger [5] initiated the study of probabilistic metric space in 1942 and by now the theory of probabilistic metric spaces has already made a considerable progress in several directions. Kutukcu et. al. [4] introduced the notion of intuitionistic Menger spaces with the help of t-norms and t-conorms as a generalization of Menger space due to Menger [5].

Definition 1.1. A t-norm is a binary operation $* : [0, 1] \times [0, 1] \to [0, 1]$ which is commutative, associative, nondecreasing for each variable and $a * 1 = a$, for all $a \in [0, 1]$.

Definition 1.2. A distance distribution function is a function $F : [0, \infty] \to [0, 1]$, that is non-decreasing and left continuous on $\mathbb{R}$, moreover, $F(0) = 0$ and $F(\infty) = 1$.
The set of all the distance distribution functions $(d.d.f.)$ is denoted by $\triangle^+$. In particular for every $x_0 \geq 0$, ε_{x_0} is the $d.d.f.$ defined by $\varepsilon_{x_0} = \begin{cases} 1 & \text{if } x > x_0, \\ 0 & \text{if } x \leq x_0. \end{cases}$

Definition 1.3. Let X be a non-empty set. A non-Archimedean Menger PM-space is an ordered triple $(X, F, *)$ where $*$ is a t-norm and F is a function from $X \times X$ into $\triangle^+$. satisfying the following conditions: $F_{x,y}(t) = 1$, $t > 0$, if and only if $x = y$; $F_{x,y}(t) = F_{y,x}(t)$; $F_{x,y}(0) = 0$ and $F_{x,y}(\max\{t, s\}) \geq F_{x,z}(t) * F_{z,y}(s)$, for all $x, y, z \in X$, $s, t \geq 0$.

*Speaker