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Isospectral Matrix Flows and Numerical Integrators on Lie Groups*

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Abstract

This paper illustrates how classical integration methods for differential equations on manifolds can be modified in order to preserve certain geometric properties of the exact flow. Runge-Kutta-Munthe-Kass method is considered and some examples are shown to verify the efficiency of the method.

Keywords: Isospectral matrix flow, Lie group, Geometric integration, Differential equation on manifold.
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1 Introduction

Isospectral matrix flows on the space of real $n \times n$ matrices M_n are characterized by the matrix differential equation

$$\frac{dA}{dt} = [A, F(A)], \quad A(0) = A_0, \tag{1}$$

where $A \in M_n$, $F : [0, \infty) \times M_n \to M_n$ is a matrix operator, $[X, Y] = XY - YX$ is the matrix commutator (also known as the Lie bracket) and A_0 is a given $n \times n$ matrix. The function A and F that obey the differential equation (1) are usually called a Lax pair. Many interesting problems can be written in this form. We just mention the Toda system, the continuous realization of QR-type algorithms, projected gradient flows, and inverse eigenvalue problems, see Chu [2] and Calvo, Iserles and Zanna [1].

Lemma 1.1. *Consider a matrix differential equation (1). Then, all eigenvalues of $A(t)$, the solution of (1), are independent of t, so that the flow (1) is isospectral flow.*

Proof. To prove the isospectrality of the flow, we define $U(t)$ by

$$\frac{dU}{dt} = -F(A(t))U(t), \quad U(0) = I_n, \tag{2}$$

*Will be presented in English
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