Semi-maximal ·-ideals in Product MV-algebras

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Abstract

In this paper, we introduce the notion of the radical of ·-ideal of PMV-algebras. We have also presented several different characterizations and many important properties of the radical of a ·-ideal in a PMV-algebra. This leads us to introduce the notion of semi-maximal ·-ideal. Finally, we show that I is a semi-maximal ·-ideal of A if and only if A/I has no nilpotent elements of A.

Keywords: PMV-algebra, ·-ideal, radical
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1 Introduction

A. Dvurecenskij and A. Di Nola in [3] introduced the notion of PMV-algebras, that is MV-algebras whose product operation $(\cdot)$ is defined on the whole MV-algebra. This operation is associative and left/right distributive with respect to partially defined addition. They showed that the category of product MV-algebras is categorically equivalent to the category of associative unital l-rings. In addition, they introduced and studied MVF-algebras [3]. They also introduced ·-ideals in PMV-algebras. Then they showed that: Any MVF-algebra is a subdirect product of subdirectly irreducible MVF-algebras [3, Corollary 5.6]. Thus they concluded that a product MV-algebra is an MVF-ring if and only if it is a subdirect product of linearly ordered product MV-algebras [3, Theorem 5.8].

In this paper, we introduce the notion of the radical of a ·-ideal in PMV-algebras. Several characterizations of this radical is given. We define the notion of a semimaximal ·-ideal in a PMV-algebra. Finally we show that A/I has no nilpotent elements if and only if I is a semi-maximal ·-ideal of A.

2 Preliminaries

In this section, we summarize properties of the basic notions in MV-algebras and PMV-algebras. For more details about these concepts, we refer the readers to [1, 3, 2].

Definition 2.1. [1] An MV-algebra is a structure $(M, \oplus, ^*, 0)$, where $\oplus$ is a binary operation, * is a unary operation, and 0 is a constant satisfying the following conditions, for any $a, b \in M$:
$(MV1)$ (M,$\oplus$,0) is an abelian monoid, $(MV2)$ $(a^*)^* = a$, $(MV3)$ $0^* \oplus a = 0^*$, $(MV4)$ $(a^* \oplus b)^* \oplus b = (b^* \oplus a)^* \oplus a$.