Topological classification of some orbit spaces arising from isometric actions on flat Riemannian manifolds

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Abstract

We give a topological classification of an orbit space $\frac{M}{G}$, arising from isometric action of a connected Lie group G on a flat Riemannian manifold M, under the conditions that the fixed point set of the action is nonempty and $\dim\frac{M}{G} \leq 3$.

Keywords: Riemannian manifold, orbit space, cohomogeneity

1 Introduction

A G-manifold is a complete differentiable manifold M with a differentiable action of a Lie group G on M. The orbit space which is the collection of all orbits $\{G(x) : x \in M\}$ will be denoted by $\frac{M}{G}$. $\dim\frac{M}{G}$ is called the cohomogeneity of M under the action of G. The most studied families of G-manifolds are cohomogeneity zero G-manifolds (also called homogeneous manifolds), for which the space of orbits consists of a single point. The topology and geometry of these spaces is for the most part well-understood. The next important family of G-manifolds are cohomogeneity one G-manifolds. Mostert proved in [9] that for a compact Lie group G, the orbit space $\frac{M}{G}$ of a cohomogeneity one G-manifold M is either a circle or interval (i.e., it is homeomorphic to S^1, $[0, 1]$, $[0, +\infty)$ or $(-\infty, +\infty)$. Mostert's theorem has been generalized for proper actions with non-compact G. Moreover, If M is endowed with a Riemannian metric, and G is a closed and connected subgroup of the isometries of M, there are more interesting results about the orbit spaces. It is proved that if M is a Riemannian manifold of negative curvature and G is a connected and closed subgroup of isometries of M, acting on M with cohomogeneity one, then the orbit space is not homeomorphic to $[0, 1]$, so by (generalized) Mostert's theorem, it would be homeomorphic to $(0, 1)$ or S^1 or R, and if in addition M is simply connected, then the orbit space is homeomorphic to $(0, 1)$ or R. This result, generalized to flat Riemannian manifolds in [7].

Theorem A. *Let M^n, $n > 2$, be a flat Riemannian manifold which is of cohomogeneity two, under the action of a connected and closed Lie group G of isometries. If $M^G \neq \emptyset$,*

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