A method of particular solutions with Chebyshev basis functions for systems of multi-point boundary value problems

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Abstract

This paper presents a new semi-analytic numerical method for solving system of multi-point boundary value problems. The method is based on the use of the particular solutions of the linearized equation. Numerical implementation confirms the validity, efficiency and applicability of the method.

Keywords: Particular solutions, System of Multi-point boundary value problems, Chebyshev basis functions.

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1 Introduction

We consider the following multi-point boundary value problem (MPBVPs):

\[ u^{(s)} = F(u, u', \ldots, u^{(s-1)}, x), \quad x \in [0, 1], \]  \hspace{1cm} (1)

\[ \sum_{j=0}^{s-1} a_{j,k} u^{(j)}(\xi_{j,k}) = d_k, \quad 0 \leq \xi_{j,k} \leq 1, \quad k = 1, \ldots, s, \]  \hspace{1cm} (2)

where some of the coefficients \( a_{j,k}, d_k \) could be equal to zero. Sometimes we write the equation in the form

\[ u^{(s)} = F(u, u', \ldots, u^{(s-1)}, x) + f(x) \]  \hspace{1cm} (3)

highlighting that \( f(x) \) does not depend on \( u \). The linear analogs of (3)

\[ u^{(s)} = \sum_{k=0}^{s-1} A_k u^{(k)}(\xi) + f(x), \quad x \in [0, 1], \]  \hspace{1cm} (4)

is also considered in the paper. We assume that \( F; A_k \) and \( f \) are smooth enough functions with respect to their arguments.

In this paper we use the semi-analytic method proposed earlier in\([1, 3, 4]\) to solve nonlinear two-point BVPs. This method is described in detail in the next section. Then we apply it to solve the system of nonlinear two-point BVPs. A numerical example illustrating the applicability of the method is placed in Section 3.

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