Independence graph of a vector space

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Abstract

Let V be a vector space over field F. The independence graph of V, denoted by Γ_V is a graph with all elements of V minus zero as vertices, and two distinct vertices v_1 and v_2 are adjacent if and only if $\{ v_1, v_2\}$ is independent. In this paper we obtain some properties of the independence graph. For example it is shown that when the independent graph is complete.

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1 Introduction

The study of algebraic structures using the properties of graphs has become an exciting research topic in the last twenty years, leading to many fascinating results and questions. There are many papers on assigning a graph to a ring, see([1]-[3]). Throughout the paper V is a vector space over a field F. We define the independence graph of V to be graph Γ_V with all elements of V minus zero as vertices, and two distinct vertices v_1 and v_2 are adjacent if and only if $\{v_1, v_2\}$ is independent.

Let Γ be a graph with vertices x and y. We define $d(x, y)$ to be the length of the shortest path from x to y. The diameter of Γ is $diam(\Gamma) = \sup\{d(x, y)|x \ and\ y\ are\ vertices\ of\ \Gamma\}$. The girth of Γ, denoted by $gr(\Gamma)$, is the length of a shortest cycle in Γ.

In Section 2, we obtain some properties of the independence graph of a vector space. Basic references for graph theory is [5]; for linear algebra see [4].

2 Main results

It is clear that the independent graph of a vector space of dimension zero is empty graph.

Theorem 2.1. *Let V be a vector space of dimension greater or equal than 1 over field F. Then Γ_V is only a set of some vertices if and only if $\dim(V) = 1$.*

Proof. First, suppose $\dim(V) = 1$. Then there is $x \in V$ such that every element of V is cx which c is an scaler. Therefore every subset of V with at least 2 elements is not independence and so there is not any edges in Γ_V.

Now, Γ_V is only a set of some vertices. Therefore for every pair of vertices x and y, there exists c such that $x = cy$. Thus $\dim(V) = 1$. □

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