On the Whisker Topology on Fundamental Group

M. Abdullahi Rashid*, S.Z. Pashaei, B. Mashayekhy, H.Torabi
Department of Pure Mathematics , Ferdowsi University of Mashhad,
P.O. Box 1159-91775, Mashhad, Iran

Abstract

In this talk, after reviewing concepts of compact-open topology, Whisker topology and Lasso toplogy on fundamental groups, we present some topological properties for the Whisker topology on a fundamental group.

Keywords: Whisker Topology, Fundamental Group, Topological Group
Mathematics Subject Classification [2010]: 57M05, 54D10, 54D15, 54H11

1 Introduction

The concept of a natural topology on the fundamental group appears to have originated with Hurewicz [8] in 1935. The topology inherited from the loop space by quotient map, where equipped with compact-open topology, on fundamental group is denoted by $\pi_1^{qtop}(X, x_0)$. Spanier [10, Theorem 13 on page 82] introduced a different topology that Dydak et al. [4] called it the Whisker topology and denoted by $\pi_1^{wh}(X, x_0)$. They also introduced a new topology on $\pi_1(X, x_0)$ and called it the Lasso topology to characterize the unique path lifting property which is denoted by $\pi_1^{l}(X, x_0)$ and showed that this topology makes the fundamental group a topological group [3]. However Biss [2] claimed that $\pi_1^{qtop}(X, x_0)$ is a topological group, but it is shown that the multiplication map is not continuous, in general, hence $\pi_1^{qtop}(X, x_0)$ is a quasitopological group (see [6]). In this talk, we show that $\pi_1^{wh}(X, x_0)$ is not a topological group, in general. In addition, it is not even a semitopological group, but it has some properties similar to toplogical groups. For instance, every open subgroup of $\pi_1^{wh}(X, x_0)$ is also a closed subgroup of $\pi_1^{wh}(X, x_0)$ and $\pi_1^{wh}(X, x_0)$ is T_0 if and only if it is T_2. Moreover, $\pi_1^{wh}(X, x_0)$ is a homogeous and regular space, and it is totally seperated if and only if is T_0.

2 Notation and Preliminaries

Definition 2.1. Let H be a subgroup of $\pi_1(X, x_0)$ and $P(X, x_0) = \{\alpha : (I, 0) \to (X, x_0) | \, \alpha \textit{ is a path}\}$ be a path space. Then $\alpha_1 \sim \alpha_2 \textit{ mod } H$ if $\alpha_1(1) = \alpha_2(1)$ and $\left[\alpha_1 * {\alpha_2}^{-1}\right] \in H$. It is easy to check that this is an equivalence relation on $P(X, x_0)$. The equivalence class of α is denoted by $\langle\alpha\rangle_H$. Now one can define the quotient space $\tilde{X}_H = \frac{P(X,x_0)}{\sim}$ and the

*Speaker