Time maps and exact multiplicity results for one-dimensional prescribed mean curvature equations

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We investigate various properties of time maps for one-dimensional prescribed mean curvature equations. Using these properties, we obtain some exact multiplicity results of positive solutions and sign-changing solutions. As it turned out, these quasilinear problems show many different phenomena from semilinear problems. Our methods are based on a detailed analysis of time maps.

1. Introduction

Consider the following quasilinear boundary value problem

\[
\begin{aligned}
-\left( \frac{u'}{\sqrt{1 + (u')^2}} \right)' &= \lambda f(u), \quad x \in (-L, L), \\
u(-L) &= u(L) = 0,
\end{aligned}
\]  

(1.1)

where \( \lambda \) and \( L \) are positive parameters. In this paper, we are concerned with exact numbers of positive solutions and sign-changing solutions of (1.1) when \( \lambda \) and \( L \) change.

The corresponding semilinear problem

\[
\begin{aligned}
-u'' &= \lambda f(u), \quad x \in (-L, L), \\
u(-L) &= u(L) = 0,
\end{aligned}
\]  

(1.2)

has been extensively studied. Many existence and multiplicity results have been obtained. For instance, see [1–6], and the references therein.

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