Nonlinear Analysis 74 (2011) 4868–4881

Contents lists available at ScienceDirect

Nonlinear Analysis

journal homepage: www.elsevier.com/locate/na

Dynamics of species in a model with two predators and one prey

Ta Viet Ton [a,*], Nguyen Trong Hieu [b]

[a] *Department of Applied Physics, Graduate School of Engineering, Osaka University, Suita Osaka 565-0871, Japan*

[b] *Faculty of Mathematics, Mechanics and Informatics, Hanoi National University of Science, 334 Nguyen Trai, Thanh Xuan, Hanoi, Viet Nam*

ARTICLE INFO

Article history:
Received 2 May 2010
Accepted 27 April 2011
Communicated by Ravi Agarwal

MSC:
34C27
34D05

Keywords:
Predator–prey system
Beddington–DeAngelis functional response
Permanence
Extinction
Periodic solution
Asymptotic stability
Liapunov function

ABSTRACT

In this paper, we study a predator–prey model which has one prey and two predators with Beddington–DeAngelis functional responses. Firstly, we establish a set of sufficient conditions for the permanence and extinction of species. Secondly, the periodicity of positive solutions is studied. Thirdly, by using Liapunov functions and the continuation theorem in coincidence degree theory, we show the global asymptotic stability of such solutions. Finally, we give some numerical examples to illustrate the behavior of the model.

1. Introduction

The dynamical relationship between predators and prey has been studied by several authors for a long time. In those researches, to represent the average number of prey killed per individual predator per unit of time, a functional, called the functional response, was introduced. The functional response can depend on only the prey's density or both the prey's and the predator's densities. However, some biologists have argued that in many situations, especially when predators have to search for food, the functional response should depend on both the prey's and the predator's densities [1–6]. One of the most popular functional responses is the fractional one as in the following prey–predator model. It is called the Beddington–DeAngelis functional response:

$$\begin{cases} x_1' = x_1(a_1 - b_1 x_1) - \dfrac{c_1 x_1 x_2}{\alpha + \beta x_1 + \gamma x_2}, \\ x_2' = -a_2 x_2 + \dfrac{c_2 x_1 x_2}{\alpha + \beta x_1 + \gamma x_2}. \end{cases}$$

In this model, $x_i(t)$ represents the population density of species X_i at time t $(i \geq 1)$; X_1 is the prey and X_2 is the predator. At time t, $a_1(t)$ is the intrinsic growth rate of X_1 and $a_i(t)$ is the death rate of X_2; $b_1(t)$ measures the inhibiting effect of the environment on X_1. This model was originally proposed by Beddington [7] and DeAngelis et al. [8] independently. Since the appearance of these two investigations, there have been many other ones for analogous systems with diffusion in a

* Corresponding author. Tel.: +81 6 6879 4249.
E-mail addresses: taviet.ton@ap.eng.osaka-u.ac.jp (T.V. Ton), hieungt@vnu.edu.vn (N.T. Hieu).

0362-546X/$ – see front matter
doi:10.1016/j.na.2011.04.061