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A continuation theorem on periodic solutions of regular nonlinear systems and its application to the exact tracking problem for the inverted spherical pendulum

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ABSTRACT

We present a continuation method to obtain a family of T-periodic solutions for a family of T-periodic systems.

In particular, we present some sufficient analytical conditions, which have the advantage of being an easy application to some systems of interest in physics or engineering. We apply these conditions to the exact tracking problem for the inverted spherical pendulum.

1. Introduction

We present a continuation method to obtain a family of T-periodic solutions $\{x_s(\cdot)\}_{0\leq s<\delta}$ associated to a family of T-periodic systems

$$\dot{x}_s(t) = F(t, s, x_s(t)), \quad \forall s \in [0, \delta[, \tag{1}$$

such that

$$x_0(\cdot) = \tilde{x}(\cdot)$$

where $\tilde{x}$ is an assigned T-periodic solution. Moreover, we give an estimate of $\|\tilde{x} - x_s\|_\infty$ with a function $\rho(s)$ which is the solution of a differential equation associated in a natural way to Jacobian matrices $\partial_x F(t, s, x)$, $\partial_s F(t, s, x)$.

This method originated from the study of the exact tracking problem for the inverted spherical pendulum and is the extension to the case of n-dimensional systems of the method presented in [1] for the case of the planar pendulum and in [2] for general two-dimensional systems, such as the nonholonomic motorcycle and the CTOL aircraft.

The use of continuation methods to prove the existence of T-periodic solutions of a differential system has been extensively considered in the literature and, in particular, as an application of the Leray–Schauder degree theory. For instance, the paper [3] proposes various topological conditions for the existence of T-periodic solutions of (1) belonging to a given bounded open set Ω of continuous periodic functions. These conditions involve the Brouwer degree of $F(t, 0, x)$ and the requirement that no T-periodic solutions of (1) belong to the boundary of Ω.

A problem related to these topological conditions lies in the fact that they depend on "a priori knowledge" of the properties of the solutions (1). In particular, checking whether conditions that no periodic trajectories of (1) lie on the boundary of Ω is in general difficult in applications. This limitation is discussed for instance in [4], which also proposes

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