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Computational analysis of rates of energy decay in elastic beams

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ABSTRACT

This paper is concerned with the damping of elastic beams of two different kinds. The first model involves the application of viscous damping at a single point either in the interior or at the boundary. The second involves a thermoelastic beam model in which mechanical damping is applied at a boundary. Since the second model is known to be uniformly stabilized via thermal effects alone, an analysis of the relative importance of the thermal and applied mechanical damping is presented. A careful analysis of the effects of rotational forces is also included using realistic model parameters.

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1. Introduction

Stabilization of structural systems is a fundamental topic in control theory. Typically, the object is to find feedback controls that damp unwanted vibrations uniformly. The control may be designed to stabilize an otherwise unstable system or increase the level of stability for a system that is already stabilized. Although damping mechanisms may be applied uniformly to the interior of the domain of a structure, damping may also be applied at a point either in the interior or on the boundary. Boundary damping in particular often affords the greatest engineering flexibility. In any event, the amount of feedback added to the system is an important issue—too much feedback (gain) may actually lead to a less stable system. A primary goal of this paper is to show computationally how variation in a gain parameter affects the damping of elastic beams by approximating the eigenvalues of the associated systems.

Two beam models are considered here. An isothermic Euler–Bernoulli beam is presented first in which linear viscous damping is applied at a single point, either on the interior or at the boundary. The object is to see how the gain affects the level of damping as the control point moves to the boundary. In particular, the computations demonstrate that adding too much feedback actually reduces the level of damping in certain modes.

The results for the Euler–Bernoulli beam are extended to a more elaborate model for a thermoelastic beam that is a one-dimensional analog of a Kirchhoff plate given in [1]. It was shown in [1] that the model is uniformly stable with mechanical damping applied to the boundary. Later, it was concluded, under a variety of model assumptions (with or without rotational forces) and boundary conditions, that the model is uniformly stable through the thermal affects alone, **without** any mechanical damping as shown in [2–4], with the most comprehensive treatment given in [5]. Similar results were obtained for a second-order thermoelastic system (the model discussed here is of fourth order) in [6,7].

The limitations of the thermal damping and the conditions that require additional damping via mechanical dissipation on the boundary have never been investigated and are indeed among the primary contributions of this paper. Additionally, some models for the thermoelastic system account for rotational forces while others do not. We provide here a numerical

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