The projective vector field of a kind of three-dimensional quasi-homogeneous system on $\mathbb{S}^2$☆

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A B S T R A C T

In this paper we study the projective vector field $Q_1$ of a three-dimensional quasi-homogeneous system with weight $(1, 1, \alpha_3)$ and degree $\delta = 2, \alpha_3 \geq 2$. Projective vector fields $Q_i$ of this kind are classified into two types. For one type, $Q_1$ has no closed orbit and at most eight singularities, which lead to a global topology of the three-dimensional system. For the other type, $Q_1$ has at most ten singularities. In addition, we show a relationship between $Q_1$ and a Lienard system of this type. For both of them we obtain some conditions for the existence of limit cycles.

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1. Introduction and statements of the main results

Let $Q(x) = (Q_1(x), Q_2(x), Q_3(x))$ be a polynomial vector field in $\mathbb{R}^3$, where $x = (x_1, x_2, x_3)$. We say that $Q$ is quasi-homogeneous with weight $(\alpha_1, \alpha_2, \alpha_3)$ and degree $\delta$ if

$$Q_1(\lambda^{\alpha_1}x_1, \lambda^{\alpha_2}x_2, \lambda^{\alpha_3}x_3) = \lambda^{\alpha_1-1+\delta}Q_1(x_1, x_2, x_3),$$

$$Q_2(\lambda^{\alpha_1}x_1, \lambda^{\alpha_2}x_2, \lambda^{\alpha_3}x_3) = \lambda^{\alpha_2-1+\delta}Q_2(x_1, x_2, x_3),$$

$$Q_3(\lambda^{\alpha_1}x_1, \lambda^{\alpha_2}x_2, \lambda^{\alpha_3}x_3) = \lambda^{\alpha_3-1+\delta}Q_3(x_1, x_2, x_3),$$

where $\lambda \in \mathbb{R}$ and $\delta, \alpha_1, \alpha_2, \alpha_3 \in \mathbb{Z}^+$ (see [1,2]). The differential system

$$\frac{dx}{dt} = Q(x)$$

is called a quasi-homogeneous polynomial system with weight $(\alpha_1, \alpha_2, \alpha_3)$ and degree $\delta$. In particular, (2) is a homogeneous polynomial system when $(\alpha_1, \alpha_2, \alpha_3) = (1, 1, 1)$.

If we transform the coordinates such that

$$x = (x_1, x_2, x_3) = (r^{\alpha_1}y_1, r^{\alpha_2}y_2, r^{\alpha_3}y_3), \quad y = (y_1, y_2, y_3) \in \mathbb{S}^2, \quad r \in \mathbb{R}^+, \quad \text{and singularities, which lead to a global topology of the three-dimensional system.}$$

then system (2) in $\mathbb{R}^3 \setminus \{(0, 0, 0)\}$ turns into

$$\frac{dr}{dt} = r^{\delta-1}(\mathbf{y}, Q(\mathbf{y})) - (\mathbf{y}, Q(\mathbf{y}))\mathbf{y}, \quad \frac{d\mathbf{y}}{dt} = r^{\delta-1}(\mathbf{y}, Q(\mathbf{y})).$$

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