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Pullback attractors for a two-dimensional Navier–Stokes model in an infinite delay case

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ABSTRACT

We prove the existence of solutions for a Navier–Stokes model in two dimensions with an external force containing infinite delay effects in the weighted space $C_\gamma(H)$. Then, under additional suitable assumptions, we prove the existence and uniqueness of a stationary solution and the exponential decay of the solutions of the evolutionary problem to this stationary solution. Finally, we study the existence of pullback attractors for the dynamical system associated to the problem under more general assumptions.

1. Introduction and statement of the problem

Navier–Stokes equations govern the motion of common fluids such as water, air, oil, etc. These equations have been the object of numerous works (e.g., see [1,2] and the references cited therein), since the first paper was published by Leray [3].

On the other hand, delay terms appear naturally, for instance as effects in wind tunnel experiments (see [4]). Very recently, Caraballo and Real [5–7] developed a full theory of the existence, stability of solutions and global attractors for Navier–Stokes models including some hereditary characteristics in several ways (fixed, variable and distributed delays). This study has been continued by some other authors; e.g., see [8–11].

While, in other fields, such as reaction–diffusion equations (see [12]), infinite-delay effects have been considered, to our knowledge they have not yet been studied so thoroughly for Navier–Stokes equations.

Our purpose is to study the following problem. Let $\Omega \subset \mathbb{R}^2$ be an open and bounded set with boundary Γ (not necessarily smooth), and consider (arbitrary) values $\tau < T$ in $\mathbb{R}$ and the following functional Navier–Stokes problem:

$$\begin{cases} \dfrac{\partial u}{\partial t} - \nu\Delta u + \displaystyle\sum_{i=1}^{2} u_i \dfrac{\partial u}{\partial x_i} = f(t) - \nabla p + g(t, u_t) \quad \text{in } (\tau, T) \times \Omega, \\ \operatorname{div} u = 0 \quad \text{in } (\tau, T) \times \Omega, \\ u = 0 \quad \text{on } (\tau, T) \times \Gamma, \\ u(\tau + r, x) = \phi(r, x), \quad r \in (-\infty, 0], x \in \Omega, \end{cases} \tag{1}$$

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