Nonlinear Analysis 74 (2011) 4898–4917

Contents lists available at ScienceDirect

Nonlinear Analysis

journal homepage: www.elsevier.com/locate/na

A geometric approach to error estimates for conservation laws posed on a spacetime

Paulo Amorim [a], Philippe G. LeFloch [b,*], Wladimir Neves [c]

[a] Centro de Matemática e Aplicações Fundamentais, Departamento de Matemática, Universidade de Lisboa, Av. Prof. Gama Pinto 2, 1649-003 Lisboa, Portugal

[b] Laboratoire Jacques-Louis Lions & Centre National de la Recherche Scientifique, Université Pierre et Marie Curie (Paris 6), 4 Place Jussieu, 75252 Paris, France

[c] Instituto de Matemática, Universidade Federal do Rio de Janeiro, C.P. 68530, Cidade Universitária 21945-970, Rio de Janeiro, Brazil

ARTICLE INFO

Article history:
Received 23 March 2011
Accepted 1 April 2011
Communicated by Ravi Agarwal

MSC:
primary 35L65
secondary 58J45

Keywords:
Hyperbolic conservation law
Flux field
Spacetime
Differential form
Entropy solution
Error estimate

ABSTRACT

We consider a hyperbolic conservation law posed on an $(N+1)$-dimensional spacetime, whose flux is a field of differential forms of degree N. Generalizing the classical Kuznetsov's method, we derive an L^1 error estimate which applies to a large class of approximate solutions. In particular, we apply our main theorem and deal with two entropy solutions associated with distinct flux fields, as well as with an entropy solution and an approximate solution. Our framework encompasses, for instance, equations posed on a globally hyperbolic Lorentzian manifold.

1. Introduction

This paper provides a general framework leading to error estimates for hyperbolic conservation laws posed on an $(N+1)$-dimensional manifold M, referred to as a spacetime and, in particular, leading to a sharp estimate for the difference, measured in the L^1 norm, between an exact solution and an approximate solution. The present paper can be regarded as a generalization to manifolds of a contribution by Bouchut and Perthame [1], who recast in a concise form the pioneering works of Kruzkov and Kuznetsov [2–4] for hyperbolic conservation laws posed on the flat (Euclidian) spacetime. We are thus interested here in extending these results to conservation laws defined on manifolds, and develop a physically more realistic setting when geometrical effects are now taken into account.

Motivated by the case of the shallow water equations on the sphere, the theory of hyperbolic conservation laws on manifolds has been developed in recent years by LeFloch together with collaborators. In particular, well-posedness results have been obtained in [5–7], and convergence results for finite volume schemes in [8,9], while an error estimate in the case of a Riemannian manifold was derived in [10]. For further results on the well-posedness theory, we also refer the reader to contributions by Panov in [11,12] and, on the finite volume schemes, to the earlier work by Coquel, Cockburn, and LeFloch [13].

Recently, in [7], LeFloch and Okutmustur introduced a framework based on differential forms and dealt with conservation laws defined on an $(N+1)$-dimensional manifold M. In their formulation, the flux of the equation is given by a field of N-forms, rather than by a vector field as was the case in earlier works. The formulation based on N-forms is geometrically

* Corresponding author.
E-mail addresses: pAmorim@ptmat.fc.ul.pt (P. Amorim), pgLeFloch@gmail.com (P.G. LeFloch), Wladimir@im.ufrj.br (W. Neves).

0362-546X/$ – see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.na.2011.04.001