Existence and properties of solutions of a control system with hysteresis effect

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ABSTRACT

We consider a control system described by two ordinary nonlinear differential equations subject to a control constraint given by a multivalued mapping with closed nonconvex values, which depends on the phase variables. One of the equations contains the subdifferential of the indicator function of a closed convex set depending on the unknown phase variable. The equation containing the subdifferential describes an input–output relation of hysteresis type.

Along with the original control constraint, we also consider the convexified control constraint and the constraint consisting of the extremal points of the convexified control constraint.

We prove the existence of solutions of our control system with various control constraints and establish certain relationships between corresponding solution sets.

1. Introduction

Consider a nonlinear control system described by two ordinary differential equations of the following form

\[
\begin{align*}
\dot{x}_1(t) &= a_1(x_1(t), x_2(t)) - a_2(x_1(t), x_2(t)) v(t) + c_1(x_1(t), x_2(t)), \\
\dot{x}_2(t) &= b_1(x_1(t), x_2(t)) v(t) + b_2(x_1(t), x_2(t)) w(t) + \partial I(x_1(t)) w(t) \ni h(x_1(t), x_2(t)) u(t) + c_2(x_1(t), x_2(t)),
\end{align*}
\]

subject to the control constraint

\[ u(t) = (u_1(t), u_2(t)) \in U(t, v(t), w(t)) \quad \text{a.e. on } T. \]

Here \( a_i(\cdot, \cdot), b_i(\cdot, \cdot), c_i(\cdot, \cdot), i = 1, 2 \), \( g(\cdot, \cdot), h(\cdot, \cdot) \) are scalar functions; for each \( v \in \mathbb{R} \), \( \partial I(x) \) is the subdifferential of the indicator function \( I(\cdot) \) of the interval \([f_-(v), f_+(v)] \subset \mathbb{R} \) with \( f_-(\cdot) \) and \( f_+(\cdot) \) being two nondecreasing functions such that \( f_- \leq f_+ \) on \( \mathbb{R} \); \( U \) is a multivalued mapping with closed bounded values; \( v_0 \) and \( w_0 \) are given numbers.

Along with (1.3) we will consider the following constraints

\[
\begin{align*}
u(t) &\in \co U(t, v(t), w(t)) \quad \text{a.e. on } T, \\
u(t) &\in \text{ext} \co U(t, v(t), w(t)) \quad \text{a.e. on } T,
\end{align*}
\]

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