Clarke coderivatives of efficient point multifunctions in parametric vector optimization

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1. Introduction

This paper is motivated by sensitivity analysis in parametric vector optimization problems. We first give some notation and definitions.

Let $f : P \times X \rightarrow Y$ be a vector function, $C : P \rightrightarrows X$ be a multifunction where $P, X$ and $Y$ are Banach spaces. Given a pointed (i.e., $K \cap (-K) = \{0\}$) closed convex cone $K \subset Y$, we consider the following parametric vector optimization problem

$$\min_k \{ f(p, x) \mid x \in C(p) \}$$

(1.1)

depending on the parameter $p \in P$. Here, $x$ is a decision variable and the cone $K$ induces a partial order $\leq_k$ on $Y$, i.e.,

$$y \leq_k y' \iff y' - y \in K, \quad y, y' \in Y.$$

(1.2)

The "min$_k$" in (1.1) is understood with respect to the ordering relation $\leq_k$ from (1.2).

We say that $y \in A$ is an efficient point of a subset $A \subset Y$ with respect to $K$ and write $y \in \text{Min}_K A$, if and only if $(y - K) \cap A = \{y\}$. If $A = \emptyset$, then we stipulate that $\text{Min}_K A = \emptyset$.

Let $F : P \rightrightarrows Y$ be a multifunction given by

$$F(p) = (f \circ C)(p) := \{ f(p, x) \mid x \in C(p) \}. $$

(1.3)

We put

$$\mathcal{F}(p) = \text{Min}_K F(p), \quad p \in P $$

(1.4)

and call $\mathcal{F} : P \rightrightarrows Y$ the efficient point multifunction of (1.1).