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Comparison results for initial value problems of second-order impulsive integro-differential inclusions☆

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ABSTRACT

In this paper, we present new maximum and minimum principles for the initial value problem for a class of second-order nonlinear impulsive integro-differential inclusions with discontinuous multivalued functions on the right hand side in a real Banach space. This allows us to lead on to the development of the comparison principle and monotone iterative technique for a nonlinear second-order impulsive integro-differential inclusion.

1. Introduction

Impulsive integro-differential equations have become more important in some mathematical models of real phenomena, especially in control, biological, medical, and informational models. Initial value problems for nonlinear integro-differential equations are used to describe a large number of nonlinear phenomena in science (see [1]). The theory of impulsive differential equations has seen considerable development in the last few decades. We mention, for instance, the monographs [1–25]. In this paper we are concerned with the following initial value problems for second-order impulsive integro-differential inclusions in Banach space E:

$$\begin{cases} x'' \in F(t, x, Tx), \quad t \in \mathcal{J}, t \neq t_k, \\ \Delta x|_{t=t_k} = -\sum_{j=1}^{k} a_{kj}x(t_j) + \sum_{j=1}^{k} b_{kj}x'(t_j), \\ \Delta x'|_{t=t_k} = -\sum_{j=1}^{k} c_{kj}x(t_j), \quad k = 1, 2, \ldots, m \\ x(0) = x_0, \qquad x'(0) = x_1, \end{cases} \tag{1}$$

where F is a multivalued map from $\mathcal{J} \times E \times E$ into 2^E, 2^E is a family of all nonempty subsets of E, $\mathcal{J} = [0, a]$ with $a > 0$, $0 < t_1 < t_2 < \cdots < t_m < a$, T is a linear operator defined by

$$(Tx)(t) = \int_0^t l(t, s)x(s)\mathrm{d}s, \quad t \in \mathcal{J},$$

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