Nonlinear Analysis 74 (2011) 2327–2341

Contents lists available at ScienceDirect

Nonlinear Analysis

journal homepage: www.elsevier.com/locate/na

Robust conjugate duality for convex optimization under uncertainty with application to data classification☆

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ARTICLE INFO

Article history:
Received 1 September 2010
Accepted 23 November 2010

MSC:
90C22
90C25
90C46

Keywords:
Robust convex optimization
Conjugate duality under uncertainty
Strong duality
Partially finite convex programs
Support vector machines
Robust Fenchel duality

ABSTRACT

In this paper we present a robust conjugate duality theory for convex programming problems in the face of data uncertainty within the framework of robust optimization, extending the powerful conjugate duality technique. We first establish robust strong duality between an uncertain primal parameterized convex programming model problem and its uncertain conjugate dual by proving strong duality between the deterministic robust counterpart of the primal model and the optimistic counterpart of its dual problem under a regularity condition. This regularity condition is not only sufficient for robust duality but also necessary for it whenever robust duality holds for every linear perturbation of the objective function of the primal model problem. More importantly, we show that robust strong duality always holds for partially finite convex programming problems under scenario data uncertainty and that the optimistic counterpart of the dual is a tractable finite dimensional problem. As an application, we also derive a robust conjugate duality theorem for support vector machines which are a class of important convex optimization models for classifying two labelled data sets. The support vector machine has emerged as a powerful modelling tool for machine learning problems of data classification that arise in many areas of application in information and computer sciences.

1. Introduction

Duality theory is a cornerstone in the area of constrained optimization and has been studied for over a century. However, real-world problems of constrained optimization often involve input data that are noisy or uncertain due to modelling or measurement errors [1–3]. Consequently, how to develop mathematical approaches that are capable of treating data uncertainty in constrained optimization has become a critical question in mathematical optimization. Over the years, various deterministic as well as stochastic approaches have been developed for treating uncertainty in optimization (see [4–10] and other references therein). In this paper, we examine a robust optimization framework [1] for studying conjugate duality theory for constrained optimization in the face of data uncertainty.

Consider the standard form convex optimization problem in the absence of data uncertainty

$$(P)\quad \inf_{x\in X} f(x),$$

☆ The authors are grateful to the referee and the Editor for their helpful comments and valuable suggestions which have contributed to the final preparation of the paper. The research was partially supported by a grant from the Australian Research Council and a Korea Science and Engineering Foundation NRL program grant of the Korea government (MEST) (No. ROA-2008-000-20010-0).

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doi:10.1016/j.na.2010.11.036