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An extension of Dickstein's "small lambda" theorem for finite time blowup

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ABSTRACT

In this paper we prove for $1 < p < 1 + \frac{1}{N+1}$, $\varphi \in L^1(\mathbb{R}^N)$ with $\int_{\mathbb{R}^N} \varphi = 0$ and $\zeta \in C_0(\mathbb{R}^N) \cap W^{1,1}(\mathbb{R}^N)$ with $\int_{\mathbb{R}^N} \zeta \neq 0$ such that $\varphi = \partial_j \zeta$ that there exists $\underline{\lambda} > 0$ such that the solution u of the equation $u_t - \Delta u = |u|^{p-1}u$ with $u(0) = \lambda\varphi$ blows up in finite time for all $0 < \lambda < \underline{\lambda}$. This extends a similar result of Dickstein who treated the case $\int_{\mathbb{R}^N} \varphi \neq 0$ and $1 < p < 1 + \frac{2}{N}$.

1. Introduction

We study here the finite-time blowup of the solutions of the following equation

$$\begin{cases} u_t - \Delta u = |u|^{p-1}u & \text{in } [0, T) \times \mathbb{R}^N \\ u(0, x) = u_0(x) \end{cases} \tag{1.1}$$

in $\mathbb{R}^N$ with $p > 1$. Given $x = (x_1, \ldots, x_N) \in \mathbb{R}^N$, we set $|x| = (x_1^2 + \cdots + x_N^2)^{1/2}$ and the first partial derivative of a function f on $\mathbb{R}^N$ with respect to the jth variable x_j is denoted by $\partial_j f$. Now let us recall that for every $u_0 \in C_0(\mathbb{R}^N)$, the Banach space of continuous functions on $\mathbb{R}^N$ that goes to zero at infinity, there exists a unique solution of (1.1) with the initial condition $u(0) = u_0$, which is defined on a maximal time interval $[0, T_{u_0})$. T_{u_0} denotes the maximal existence time of the solution u with corresponding initial value u_0, i.e. $u \in C([0, T_{u_0}), C_0(\mathbb{R}^N))$. If $T_{u_0} < \infty$, then the solution blows up in finite time, i.e. $\|u(t)\|_{L^\infty} \to \infty$ as $t \to T_{u_0}$, and if $T_{u_0} = \infty$ then the solution is called global in time. The main result of this paper is the following.

Theorem 1. *Let $1 < p < p_N = 1 + \frac{1}{N+1}$, and let $\varphi \in L^1(\mathbb{R}^N) \cap C_0(\mathbb{R}^N)$ satisfy $\int_{\mathbb{R}^N} \varphi = 0$ and let $\zeta \in C_0(\mathbb{R}^N) \cap W^{1,1}(\mathbb{R}^N)$ satisfy $\int_{\mathbb{R}^N} \zeta \neq 0$ such that $\varphi = \partial_j \zeta$.*

Given $\lambda > 0$ let u_λ be the solution of (1.1) corresponding to $u_0 = \lambda\varphi$. Then there exists $\underline{\lambda}$ such that u_λ blows up in finite time for all $0 < \lambda < \underline{\lambda}$.

This result is motivated by a recent paper of Dickstein [1], but the question goes back to Fujita's classical result [2] in 1966. Indeed, Fujita proved that if

$$p < p_F = 1 + \frac{2}{N}$$

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