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On an asymptotically p-linear p-Laplacian equation in $\mathbb{R}^N$ ☆

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ABSTRACT

In this paper, we study the existence of positive solutions and sign-changing solutions for the p-Laplacian equation

$$-\Delta_p u + \lambda g(x) \mid u\mid^{p-2} u = f(x, u), \quad x \in \mathbb{R}^N,$$

where λ is a positive parameter and the nonlinear term f is superlinear at zero and asymptotically p-linear at infinity.

1. Introduction

In this paper we are concerned with the following p-Laplacian equation in $\mathbb{R}^N$:

$$-\Delta_p u + \lambda g(x)|u|^{p-2}u = f(x, u), \quad x \in \mathbb{R}^N, \tag{1.1}$$

where $\Delta_p u = \mathrm{div}(|\nabla u|^{p-2}\nabla u)$ is the p-Laplacian, $1 < p < N$, λ is a positive parameter. Assume that g satisfies the condition

(G) $g \in L^\infty(\mathbb{R}^N)$, $g \geq 0$, $\lim_{|x|\to\infty} g(x) = 1$ and meas $\{x \in \mathbb{R}^N : g(x) < 1\} > 0$.

As for the nonlinear term f, we make the following assumptions.

(f_1) $f(x, t) \in C(\mathbb{R}^N \times \mathbb{R}, \mathbb{R})$, $f(x, 0) = 0$.
(f_2) $\lim_{|t|\to 0} \frac{f(x,t)}{|t|^{p-2}t} = 0$ uniformly in $x \in \mathbb{R}^N$.
(f_3) There exists α such that $\lim_{|t|\to\infty} \frac{f(x,t)}{|t|^{p-2}t} = \alpha$ for a.e. $x \in \mathbb{R}^N$.
(f_4) $\overline{\lim}_{|x|\to\infty} \sup_{t\neq 0} \frac{f(x,t)}{|t|^{p-2}t} = \beta < \lambda$.
(f_5) There exists $C > 0$ such that $|f(x, t)| \leq C|t|^{p-1}$ for a.e. $x \in \mathbb{R}^N$ and all $t \in \mathbb{R}$.

Remark 1.1. Under the conditions (f_1)–(f_5). It is easy to verify that $\alpha \leq \beta \leq C$. Strict inequalities may happen. Here is an example.

$$f(x, t) = \frac{|t|^{p-2}t^3}{a(t) + t^2} + \frac{|t|^{p-2}t^3 e^{1-t^2}(1 + |x|^2)}{1 + 2|x|^2},$$

where $a(t) = 1$ for $t < 1/2$, $a(t) = 2 - 2t$ for $t \in [1/2, 1]$ and $a(t) = 0$ for $t > 1$. For such a function f, we have $\alpha = 1$, $\beta = 3/2$ and $C = 2$.

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