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A Schrödinger equation with time-oscillating critical nonlinearity☆

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ABSTRACT

In this paper, we study the time-oscillating critical nonlinear Schrödinger equation $\mathrm{i}u_t + \Delta u + \theta(\omega t)|u|^{\frac{4}{n-2}}u = 0$ in $\mathbb{R}^n$ $(n \geq 3)$, where θ is a periodic function. We show that, for a given initial condition $u(0) = \varphi$ in H^1, the solution u_ω converges as $|\omega| \to \infty$ to the solution U of the limiting equation $\mathrm{i}U_t + \Delta U + I(\theta)|U|^{\frac{4}{n-2}}U = 0$ with the same initial condition, where $I(\theta)$ is the average of θ. We also show that if the solution U is global and has a certain decay property, then u_ω is also global if $|\omega|$ is sufficiently large. Similar results for the subcritical problem are given by Cazenave and Scialom (2010) [9].

1. Introduction

In this article, we consider the following critical nonlinear Schrödinger equations in $\mathbb{R}^n$ $(n \geq 3)$:

$$\begin{cases} \mathrm{i}u_t + \Delta u + \theta(\omega t)|u|^\alpha u = 0, \\ u(0) = \varphi \in H^1(\mathbb{R}^n), \end{cases} \tag{1.1}$$

and their equivalent integral forms

$$u(t) = \mathrm{e}^{\mathrm{i}t\Delta}\varphi + \mathrm{i}\int_0^t \mathrm{e}^{\mathrm{i}(t-s)\Delta}\theta(\omega s)|u(s)|^\alpha u(s)\mathrm{d}s, \tag{1.2}$$

where $(\mathrm{e}^{\mathrm{i}t\Delta})_{t\in\mathbb{R}}$ is the Schrödinger group, $\alpha = \frac{4}{n-2}$, and $\theta \in C^1(\mathbb{R}, \mathbb{R})$ is a τ-periodic function for some $\tau > 0$ and $\omega \in \mathbb{R}$. Such equations originate from physics. For example, [1,2] investigated the effect of a time-oscillating term in a factor of the nonlinearity in nonlinear Schrödinger equations. In particular, Abdullaev et al. [1] investigated solutions which are global for large frequencies, while Konotop and Pacciani [2] studied solutions which blow up in finite time.

To begin with, we recall some classical results about the initial value problem associated with critical nonlinear Schrödinger equations on the whole space $\mathbb{R}^n$. For θ a constant, in [3] and references therein, the authors showed that the solution of (1.1) has local well-posedness. For our case, remarking that the function θ is uniformly bounded, we only need to take the L^∞ norm of θ when the nonlinearity has to be estimated in some norms. Keeping this in mind and applying the method in [3] (Theorem 2), one can show the local well-posedness of (1.1). For the sake of conciseness, we only state the results without detailed proof. (Here and in the rest of the paper, a pair (q, r) is called admissible if $2 \leq r \leq 2n/(n-2)$ $(2 \leq r < \infty$ if $n = 1, 2)$ and $2/q = n(1/2 - 1/r)$.)

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