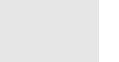
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Brake type closed characteristics on reversible compact convex hypersurfaces in \mathbf{R}^{2n}

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ABSTRACT

In this paper, let Σ be a C^3 compact convex in \mathbf{R}^{2n} satisfying the reversible condition $N\Sigma = \Sigma$ with $N = \text{diag}(-I_n, I_n)$. We prove that if there are exactly *n* geometrically distinct closed characteristics on Σ and all of them are nondegenerate, then all of them must be brake orbits up to a suitable translation of time. Moreover, for n = 2 or 3, we prove that if there are exactly *n* geometrically distinct closed characteristics on Σ , then all of them must be brake orbits up to a suitable translation of time.

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1. Introduction and main results

In this paper, let $J = \begin{pmatrix} 0 & -l \\ l & 0 \end{pmatrix}$ and $N = \begin{pmatrix} -l & 0 \\ 0 & l \end{pmatrix}$ with *I* being the identity in \mathbb{R}^n . Let Σ be a C^3 compact hypersurface in \mathbb{R}^{2n} bounding a bounded and strictly convex domain *C* in \mathbb{R}^{2n} . Without loss of generality, we may assume that *C* contains the origin. We denote the set of all such hypersurfaces in \mathbb{R}^{2n} by $\mathcal{H}(2n)$. We call Σ reversible if it satisfies the reversible condition $\Sigma = N\Sigma := \{Nx | x \in \Sigma\}$ and denote by $\mathcal{H}_b(2n)$ the set of all reversible hypersurfaces in $\mathcal{H}(2n)$. We consider the dynamics problem of finding $\tau > 0$ and an absolutely continuous curve $x : [0, \tau] \to \mathbb{R}^{2n}$ such that

$$\dot{x}(t) = JN_{\Sigma}(x(t)), \quad x(t) \in \Sigma,$$

$$x(\tau + t) = x(t), \quad \forall t \in \mathbf{R},$$
(1.1)
(1.2)

where $N_{\Sigma}(x)$ is the outward unit vector at *x* of Σ .

A solution (τ, x) of problem (1.1)–(1.2) is called a closed characteristic on Σ . Furthermore, as in [1], if the closed characteristic (τ, x) satisfies that x(-t) = Nx(t) for all $t \in \mathbf{R}$, we call it a brake orbit on Σ .

Two closed characteristics (τ_1, x_1) and (τ_2, x_2) are called geometrically distinct if $x_1(\mathbf{R}) \neq x_2(\mathbf{R})$. We denote by $\mathcal{J}(\Sigma)$ and $\tilde{\mathcal{J}}(\Sigma)$ the set of all closed characteristics (τ, x) on Σ with τ being the minimal period of x and geometrically distinct ones on Σ respectively. For $(\tau, x) \in \mathcal{J}(\Sigma)$, we denote by $[(\tau, x)]$ the set of all elements in $\mathcal{J}(\Sigma)$ which are geometrically the same as (τ, x) .

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