



Strong convergence of a general iteration scheme in $CAT(0)$ spaces

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ABSTRACT

We introduce and study strong convergence of a general iteration scheme for a finite family of asymptotically quasi-nonexpansive maps in convex metric spaces and $CAT(0)$ spaces. The new iteration scheme includes modified Mann and Ishikawa iterations, the three-step iteration scheme of Xu and Noor and the scheme of Khan, Domlo and Fukhar-ud-din as special cases in Banach spaces. Our results are refinements and generalizations of several recent results from the current literature.

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1. Introduction and basic definitions

Let T be a self-map on a nonempty subset K of a metric space (X, d) . Denote by $F(T) = \{x \in K : T(x) = x\}$ the set of fixed points of T .

The map T is said to be: (i) uniformly L -Lipschitzian if for $L > 0$, we have $d(T^n x, T^n y) \leq L d(x, y)$ for $x, y \in K$, and $n \geq 1$; (ii) asymptotically nonexpansive [1] if there exists a sequence $\{k_n\} \subset [1, \infty)$ with $\lim_{n \rightarrow \infty} k_n = 1$ such that $d(T^n x, T^n y) \leq k_n d(x, y)$ for $x, y \in K$, and $n \geq 1$; and (iii) asymptotically quasi-nonexpansive if $F(T) \neq \emptyset$ and there exists a sequence $\{k_n\} \subset [1, \infty)$ with $\lim_{n \rightarrow \infty} k_n = 1$ such that $d(T^n x, p) \leq k_n d(x, p)$ for $x \in K$, $p \in F(T)$, and $n \geq 1$.

If $k_n = 1$ for $n \geq 1$ in the above definitions (ii), (iii), then T becomes a nonexpansive and a quasi-nonexpansive map, respectively.

Various iteration processes have been studied for an asymptotically nonexpansive map T on a convex subset K of a normed space E . Schu [2] considered the following modified Mann iterations:

$$x_{n+1} = (1 - a_n)x_n + a_n T^n x_n, \quad n \geq 1, \quad (1.1)$$

where $0 < a_n < 1$.

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